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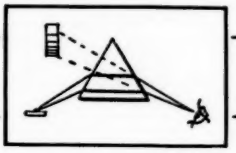

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THE OPTICAL SOCIETY OF AMERICA was organized in 1916 "*to increase and diffuse the knowledge of optics in all its branches, pure and applied, to promote the mutual interests of investigators of optical problems, of designers, manufacturers and users of optical instruments and apparatus of all kinds and to encourage cooperation among them.*" It aims to serve the interests of all who are engaged in any branch of optics from fundamental research to the manufacture of optical goods.

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The Society holds two meetings per year: An annual meeting in the autumn and a midwinter meeting jointly with the American Physical Society.

All changes of address of members of the Optical Society of America should be communicated to the Secretary, Professor Arthur C. Hardy, Massachusetts Institute of Technology, Cambridge, Massachusetts.

Journal of the
OPTICAL SOCIETY *of* AMERICA

VOLUME 30 MAY, 1940 NUMBER 5

Editorial Comment

QUARTER-CENTURY CELEBRATION

THOUGH optics in America is still young and vigorously growing, the Optical Society of America is already making plans to hold its twenty-fifth annual meeting. An active and able program committee is preparing to make this meeting, to be held next fall, of outstanding interest and importance, and a large attendance is expected.

Rochester, New York, has been selected as the meeting place, and October 3, 4 and 5 are the dates. As the home of the University of Rochester (with its celebrated Institute of Applied Optics), of the Eastman Kodak Company, and of the Bausch and Lomb Optical Company, as well as of other leading optical manufacturers, Rochester is one of the most active centers in American and world optics, and will give a welcome to members and friends of the society which should make this anniversary meeting a memorable occasion. It was from a Rochester association of scientific workers interested in optics that the Optical Society of America originated.

An unusual industrial exhibit of optical instruments and accessories will be one of the features of the meeting. It is expected that this exhibit may be so arranged as to illustrate the advances in optics which have been made during the lifetime of the Society.

The Rochester meeting will be the occasion of the bestowal, for the first time, of the recently-created Adolph Lomb medal, which is "to be awarded, not oftener than once in two years, to a person under thirty years of age for noteworthy contribution to optics." Adolph Lomb, in whose name this medal was established, was a charter member of the Society, and served as its first Treasurer for sixteen years until his death in 1932.

Twelve of the charter members of the Optical Society are still active in its rolls. Their names follow:

Ames, Adelbert, Jr.	Mees, C. E. Kenneth
Bausch, Edward	Nutting, P. G.
Frederick, C. W.	Rayton, Wilbur B.
Gage, H. Phelps	Southall, J. P. C.
Ives, Herbert E.	Tillyer, Edgar D.
Jones, Loyd A.	Wright, Fred E.

President Gibson has suggested that the twenty-fifth annual meeting might well be the occasion for a stimulation in growth of the society. A very active membership committee is now at work under the chairmanship of Miss Dorothy Nickerson, but every member of the society is empowered to attempt to convince any of his or her friends who are interested in optics that they also should become members. A comparatively small growth in membership will bring great benefits, making possible publication of a larger and better journal as well as stimulating optical activity in America. President Gibson has emphasized that to aid in the growth of the society may well be considered a privilege rather than a duty. Duty or privilege, it is certain that growth in membership will redound to the benefit of the society as a whole and of each member individually.

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Operating Temperatures of Vapor Lamps*

J. W. Marden, N. C. Beese and George Meister
Research Laboratory, Westinghouse Lamp Division, Bloomfield, New Jersey
(Received March 20, 1940)

A GAS or vapor in a state of discharge contains various particles each having different velocities, and according to the kinetic theory, different temperatures. Hence an arc cannot be assigned one definite temperature value. Pirani and Rompe[1] report the temperatures in a neon discharge at 50 ma per cm² as follows: Electron temperature, 15,000°C; excited atoms temperature, 10,000°C; ionized atoms temperature, 12,000°C; neutral atoms temperature and tube walls, 50°C. Although the temperature of the walls of Pirani's lamp and the neutral atoms is 50°C, the ions may have temperatures of many thousands of degrees.

An average temperature value of a discharge has little significance where the arc stream is constricted away from the walls of the container. A commercial 400-watt mercury arc has an average temperature of 5000 or 6000°K in the center of the arc stream[2] while the temperature near the glass walls is about 400°C.

For the sake of simplicity and also due to the fact that the temperature of the walls of a discharge tube often controls the vapor pressure, which in turn determines the electrical and luminous characteristics of an arc, it is customary to consider the wall temperature as the "operating temperature."

Determination of Operating Temperature

Low pressure mercury discharges have been immersed in water during tests that covered the temperature range of 0°C to nearly 100°C. The temperature of the water was taken as the operating temperature of the lamps.[3] Higher temperatures, up to 300° or 400°, were determined by fine wire thermocouples fastened onto various parts of the bulb, usually by kaolin and water glass. DeGroot[4] attached to an experimental high pressure mercury arc lamp a small-bore closed-end manometer containing neon gas trapped behind a mercury column. The compression of the neon gas gave values for the pressures that in turn could be converted to the corresponding temperatures.

Bulb Temperature and Operating Conditions

Data on the variations of the temperature of different sections of the walls of mercury arcs with changes in operating conditions are given in graphical form in Figs. 1 and 2.

It is evident that a discharge must be carefully designed for a specified energy input and a particular operating condition, such as burning in air or in a vacuum housing, if temperature distribution over the bulb surface is approximately uniform. With a more or less uniform temperature distribution the relationship between bulb temperature and watts per cm² of bulb surface can be shown diagrammatically as in Fig. 3.

Inert Gas in Metal-Vapor Lamps

Practically all vapor lamps used for illumination contain inert gas such as argon or neon.

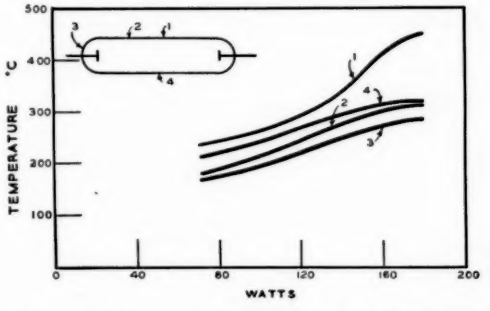

Fig. 1. Temperature distribution along walls of experimental mercury arc burned in horizontal position. Thermocouple positions are indicated.

* Presented at the American Institute of Physics Temperature Symposium, New York, New York, November 2, 1939.

[1] M. Pirani and R. Rompe, Trans. Electrochem. Soc. **69**, 417 (1936).

[2] W. Elenbaas, Physica **1**, 211 (1934).

[3] Marden, Beese and Meister, Trans. I.E.S. **34**, 55 (1939).

[4] W. DeGroot, De Ingenieur **50**, E92 (1935).

The inert gas facilitates starting at low voltage, and also materially affects the temperatures at which low pressure resonance radiation lamps are best operated. The effect of adding inert gas on the characteristics of low pressure lamps is in many cases the same as raising the temperature. DeGroot's data[5] is reproduced in Fig. 4 for a bulb-type sodium lamp in vacuum housing. Neon gas materially improves the efficiency of this lamp. In a low pressure mercury discharge the efficiency varies with the kind and pressure of inert gas as depicted in Fig. 5. These curves show the response of a fluorescent coating which indicates the production of resonance radiation at 2537A. During operation the characteristics of high pressure discharges are not affected by the quantities of inert gas used in lamps.

Production of Maximum Amounts of 2537A Radiation

The fluorescent lamp and the Sterilamp depend upon the mercury resonance radiation of wavelength 2537A for their operating efficiencies. This particular radiation is generated most efficiently at approximately room temperature. It was discovered, however, that a pronounced decrease in efficiency occurred when mercury resonance lamps were operated either at freezing or elevated temperatures. Immersing the lamps in water served as a simple expedient for keeping the bulb temperature constant. The data show that the intensity of both the fluorescent light and the mercury resonance radiations are low near 0°C, increase to a maximum between 40°C and 45°C and then decrease again above 60°C. If the assumption is made that the temperature of the water is the same as the temperature of the neutral mercury vapor atoms within the lamps, the temperature of maximum output corresponds to a vapor pressure of about 0.01 mm mercury. The relationship between temperature, vapor pressure and the mercury resonance line at 2537A is given in Fig. 6.

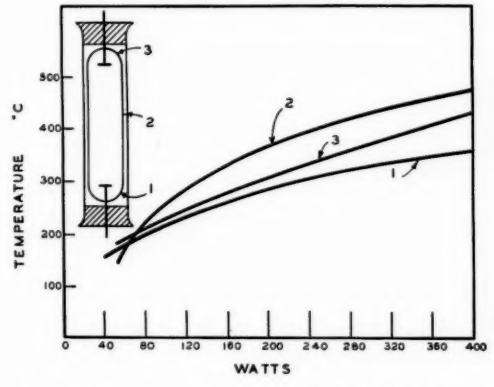

Fig. 2. Temperatures at three indicated points on walls of a mercury arc lamp burned vertically inside a closed glass housing.

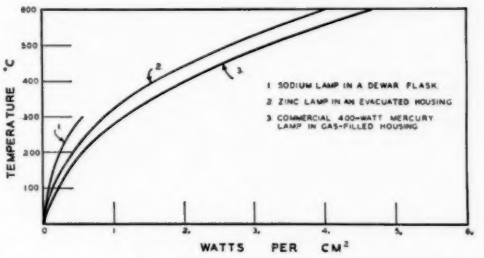
TEMPERATURE °C
WATTS PER CM²
1 SODIUM LAMP IN A DEWAR FLASK
2 ZINC LAMP IN AN EVACUATED HOUSING
3 COMMERCIAL 400-WATT MERCURY LAMP IN GAS-FILLED HOUSING

Fig. 3. Bulb temperature and watts per cm² of bulb surface for lamps burned under various operating conditions.

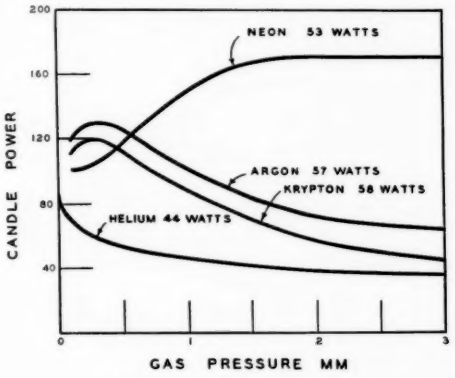

Fig. 4. Effect of inert gas pressure on light output of a sodium lamp.

Maximum Resonance Radiation of Sodium Vapor

Uyterhoeven and Verburg[6] state that the best operating temperature of a sodium lamp is about

[5] W. DeGroot, U. S. Patent 1,919,933, July 25, 1933.

[6] W. Uyterhoeven and C. Verburg, Comptes rendus, **200**, 536 (1935).

240°C while Found[7] gives a value of 230°C. Pirani,[8] however, operated sodium lamps of extremely high efficiencies at 300 to 350°C. According to Fonda and Young[9] the operating temperatures of sodium lamps (Fig. 7) must be controlled within reasonably close limits. The shape of the upper curve in Fig. 7 for the yellow resonance lines of sodium is very similar to the curve in Fig. 6 for the 2537A line of mercury.

Effect of Temperature on Spectral Distribution of Vapor Lamps

Pronounced changes occur in the radiations emitted by vapor lamps as the operating temperatures and corresponding pressures are varied. The visible light from a low temperature, low pressure mercury discharge is confined to a few sharp spectral lines containing only a small percentage of the radiated energy. Under such conditions approximately 50 percent of the total energy is located in the ultraviolet resonance line of wave-length 2537 angstroms. As the operating temperature is increased, more of the 2537 radiation is converted into the longer wave-length ultraviolet and visible lines. With an increase of bulb temperature to 360°C the pressure is increased to about an atmosphere and the efficiency is increased from 4 to 40 lumens per watt. The temperature of the inner wall of the water-cooled quartz capillary arc is of the order of 800°C and the luminous efficiency has increased to about 65 lumens per watt.

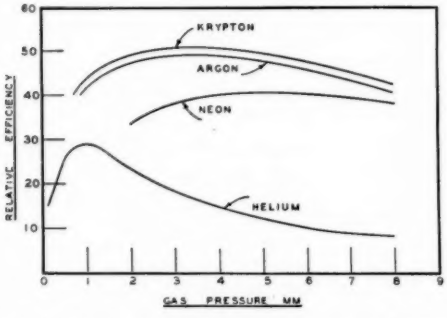

Fig. 5. Effect of inert gases at low pressures on fluorescent lamps burned at constant current.

Data showing the relationship between tube diameter, operating pressure and bulb temperature for a constant energy input of 20 watts per cm of arc stream is given in Table I, taken from Elenbaas.[10]

According to DeGroot[11] marked changes occur in the ultraviolet region as the operating temperature and pressure of a quartz mercury lamp are increased from a value of less than one atmosphere to 100 atmospheres. At the lowest

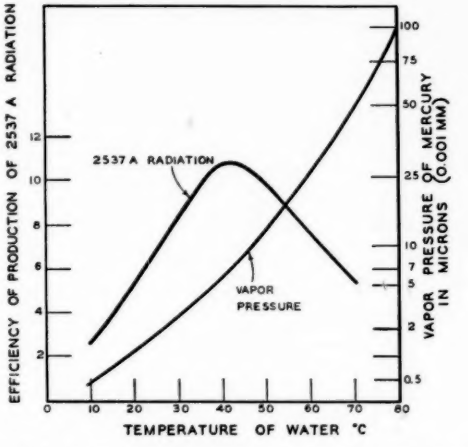
EFFICIENCY OF PRODUCTION OF 2537 A RADIATION
2537 A RADIATION
VAPOR PRESSURE
VAPOR PRESSURE OF MERCURY IN MICRONS (0.001 MM)
TEMPERATURE OF WATER °C

Fig. 6. Relation between temperature, vapor pressure and production of mercury 2537Å radiation.

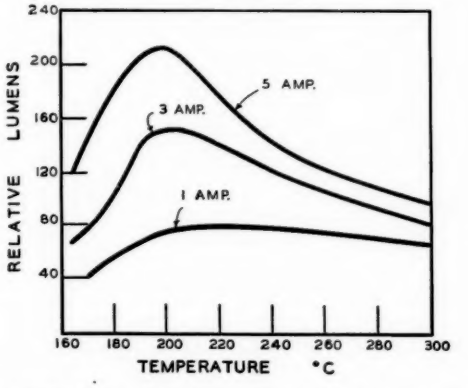

Fig. 7. Variation of luminous output with temperature of a sodium lamp.

pressures only sharp spectral lines occur, but with higher pressures and temperatures the lines become broad and fuzzy, and have a continuous

[7] C. G. Found, Gen. Elec. Rev. **37**, 269 (1934).
[8] M. Pirani, Zeits. f. tech. Physik **11**, 482 (1930).
[9] G. R. Fonda and A. H. Young, J. Opt. Soc. Am. **24**, 31 (1934).
[10] W. Elenbaas, Physica **2**, 787 (1935).
[11] W. De Groot, reference 4.

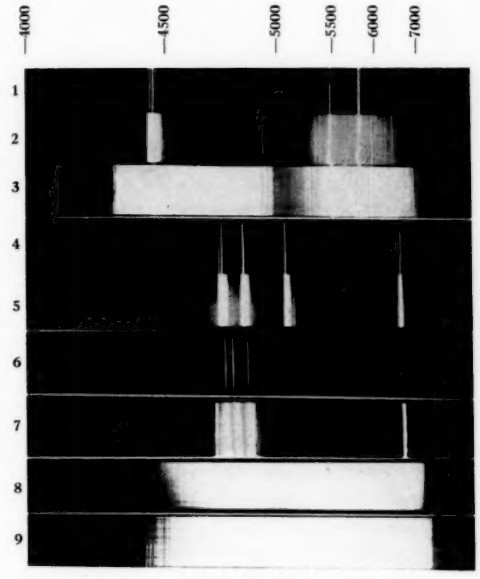

FIG. 8. Spectrograms of various metallic vapor arcs. 1, 2, 3 low, intermediate and high pressure mercury; 4, 5 low, high pressure cadmium; 6, 7 low, high pressure zinc; 8 tellurium; 9 solar spectrum.

spectrum superimposed over them. In addition, the 2537A line becomes an absorption line that extends to the long wave side for a distance of about 100A at a pressure of 100 atmospheres.

Changes occur in the visible region that parallel the shifts in spectral energy distribution found in the ultraviolet, i.e., broad fuzzy lines with a superimposed continuous background. This continuous spectrum modifies the color of the mercury lamp to produce a whiter light. The mercury spectrograms in Fig. 8 show the differences between low and high temperature lamps.

Extremely high temperature and pressure mercury arcs have been described by Bol.[12] A quartz capillary lamp is surrounded by water which is circulated at a pressure of 1000 atmospheres to prevent explosion of the lamp envelope. Calculations show that the temperature on the inside wall surfaces of this lamp must be about 1300°C. Elenbaas[13] calculated the temperature in the arc stream of a water-cooled lamp operating at a pressure of 200 atmospheres to be 8900°K. Under these conditions the lamp has a brightness about equal to that of the sun.

[12] C. Bol, Phys. Rev. **53**, 214(A) (1938).
[13] W. Elenbaas, Zeits. f. tech. Physik **17**, 61 (1936).

Low pressure glass cadmium and zinc vapor lamps are operated at a temperature of about 300°C.[14] The vapor pressure at this temperature is only a few microns. High pressure lamps are made with quartz envelopes to withstand a temperature of operation of about 700°C. Some of these lamps have been operated at pressures above one atmosphere. High temperature operation of both cadmium and zinc lamps increases the efficiency by a factor of 4 or 5 and also shifts a larger percentage of the total light into the red spectrum lines. The spectrograms in Fig. 8 show the broad fuzzy lines radiated by these high temperature discharges.

At low temperatures, pressures and efficiencies the light from a tellurium vapor lamp has a continuous spectrum and approximates the spectral energy distribution of a blackbody at 6000°K, while at higher operating temperatures the light becomes predominately yellow or gold in color. Under such conditions the spectral energy is more like that of a candle flame than a very highly heated incandescent body.[15] The tellurium arc is compared to the solar spectrum at the bottom of Fig. 8.

Caesium vapor was found by Pirani[16] and his associates to give a nearly continuous spectrum throughout the visible range at a temperature of about 300°C and a few mm pressure. Mohler[17] calculated a theoretical efficiency of the order of 180 lumens per watt for vapor pressures of 176 mm or an operating temperature of about 550°C.

OPERATING TEMPERATURE RANGES OF VAPOR LAMPS

Table II gives the approximate operating temperatures of commercial vapor lamps. Experi-

TABLE I. *Temperature of high pressure mercury arcs.*

TUBE DIAM. IN MM	VAPOR PRESSURE IN MM	INSIDE WALL TEMPERATURE
40	500	600°K
20	1000	740
10	2000	905
4.5	4000	1060
2	10000	1320

[14] Marden, Beese and Meister, Trans. I.E.S. **32**, 84 (1937).
[15] Marden, Beese and Meister, J. Frank. Inst. **225**, 45 (1938).
[16] Lax, Pirani and Rompe, Naturwiss. **25**, 393 (1935).
[17] F. L. Mohler, J. Opt. Soc. Am. **29**, 152 (1939).

TABLE II. *Operating ranges of commercial lamps.*

MERCURY LAMPS	WATTS	APPROXIMATE OPERATURE TEMPERATURE	APPROX. PRESSURE
Sterilamp and Fluorescent	15–40	20–48°C	1–10 microns
H-2 Glass	250	300	400 mm
H-1 Glass	400	360–400	1 atmos.
H-5 Quartz	250	470	5 atmos.
H-4 Quartz	100	525	10 atmos.
H-3 Quartz	85	590	20 atmos.
H-6 Quartz (water cooled)	1000	785	100 atmos.
Experimental (water cooled)	2500	(1300)	1000 atmos.
Sodium Lamps			
Sodium	200	220–265	0.5 to 5 microns

mental lamps have been operated over a large range of temperatures. Low pressure cadmium and zinc lamps are operated at about the same temperature and under the same conditions as the sodium vapor lamp. High pressure cadmium and zinc lamps are operated in quartz up to and even above atmospheric pressure for a short time but such lamps have poor life.

The tellurium vapor lamp can be operated at 100 or 200 mm pressure in quartz. Although such lamps have been operated above one atmosphere pressure, the life is only of the order a few minutes under such conditions.

If some substance useful for lamp bulb construction could be produced which would withstand higher temperatures than quartz many discharges could be operated at greater brightnesses and greater efficiencies.

The Carbon Arc as a Radiation Standard*

H. G. MacPherson
National Carbon Company, Edgewater Works, Cleveland, Ohio
(Received March 3, 1940)

THE positive crater of the pure carbon arc provides a source of radiation that has desirable qualities for many technical applications. Some of the characteristics that make the arc an attractive light source for certain exacting uses are:

(1) It provides a source of reproducible intensity and spectral energy distribution. This reproducibility is independent of time, since it depends only on the physical properties of a pure element. Thus intensity measurements made by Otto Lummer[1] in 1914 are still readily intelligible today despite possible changes in light units and radiation constants, because they are expressed on a scale relative to the brightness of a carbon arc.

(2) The reproducibility of the source can be maintained regardless of errors in current and voltage measuring devices. The proper current to obtain uniform brightness can be adjusted merely by watching an image of the arc, without an ammeter in the circuit.

(3) The arc provides a much more intense source of continuous radiation than the tungsten lamp. The continuous spectrum is strong enough so that it may be used to obtain absorption spectra in the ultraviolet photographically, as well as to provide a good source for the visible and infra-red.

The uses to which the solid carbon arc has been put include the calibration of optical pyrometers, and the provision of a source of a continuous spectrum for absorption spectrum work and for the determination of high flame temperatures by the spectrum line reversal method. Its constant spectral energy distribution characteristic has made it useful as an intensity standard for evaluating the intensity of emission spectra, and as a standard light source for color work.

The constancy of the maximum brightness of the carbon arc is a particularly interesting attribute, making possible its use as a calibration source for optical pyrometers. A type of arc especially suitable for this application will be described and modifications of it for other purposes will then be discussed.

In 1935 Chaney, Hamister and Glass[2] reported an investigation on the various factors affecting the operation of an arc suitable as a calibration standard for optical pyrometers, and they specified conditions under which an arc can be obtained that will give a reproducible maximum temperature of the positive crater.

For a positive electrode, the arc employs a spectroscopically pure graphite rod of $\frac{1}{4}$ inch or less diameter. Any type of negative electrode that gives a steady arc may be used, provided it contains no impurities that will emit appreciable radiation in the arc stream. A spectroscopically pure graphite rod $\frac{1}{8}$ inch or less in diameter is recommended, or a $\frac{3}{16}$-inch carbon rod of spectroscopic purity can be used.

These are the carbons known to the trade as special spectroscopic carbons, and should not be confused with the ordinary spectroscopic carbons frequently used for less exacting work. The positive electrode is supported horizontally, while the negative is held vertically in the same plane with its tip just below the level of the positive electrode and about $\frac{1}{4}$ inch in front of it. Provision must be made, of course, for each electrode to be advanced along the direction of its axis as it is consumed in the arc. Various microscope arc lamps are available that can be adapted for burning such an arc.

The maximum current that can be forced through the arc without hissing should be provided. This is from 10 to $12\frac{1}{2}$ amperes with the $\frac{1}{4}$-inch graphite positive electrode. The arc voltage will be between 60 and 70 volts. The power for this can be taken from a regular

* Presented at the American Institute of Physics Temperature Symposium, New York, New York, November 2, 1939.

[1] O. Lummer, *The Liquefaction of Carbon and Attainment of the Solar Temperature* (Braunschweig, 1914).

[2] N. K. Chaney, V. C. Hamister and S. W. Glass, Trans. Electrochem. Soc. **67**, 201 (1935).

110-volt d.c. supply line with a suitable variable ballast resistance in series with the arc. The ballast resistance is necessary to maintain arc stability. Also, the steadier the line voltage, the steadier will be the arc.

Just after the arc is started it will hiss and sputter considerably. However, after a few minutes have elapsed, during which the graphite becomes heated up, the arc will burn quietly. Then, as the current is increased, the brightly incandescent area expands until it covers the end of the electrode. At currents greater than that at which the arc crater is of nearly uniform brightness, the arc will suddenly break out in a hissing or sputtering noise. This is accompanied by a sudden increase in current and a drop in arc voltage. The condition is called "overload," and is to be avoided. Once hissing has started, a quiet arc can be restored by temporarily decreasing the arc current, following this by a cautious increase to the optimum value. The preferred operating current is one slightly below the overload point, at which current the brightly incandescent region just covers the end of the electrode. The exact current at which this occurs will vary with the degree to which the electrode spindles. The amount of spindle, caused by the oxidation of the graphite before it reaches the arc, will vary with different conditions of burning the arc, so that it is impossible to specify the optimum current exactly.

The relative position of the positive and negative tips also affects the steadiness of the arc. However, it is easy to determine and hold a relationship in which the face of the positive electrode burns off squarely. This can be done best by projecting an enlarged side image of the arc onto a wall by means of a simple lens. The image will show the two white-hot glowing electrodes with a violet-colored intensely hot arc stream bridging the angle between them. The violet color of the arc stream is due entirely to cyanogen band radiation which extends intermittently over the spectral region between 3500A and 4200A. There is no appreciable arc stream radiation in the blue, green, or red regions of the spectrum. Consequently, pyrometric observations of the positive crater may be made through a section of the arc stream without being affected by it.

When the arc is operated under these conditions, the tip of the positive crater is maintained at the sublimation temperature of carbon. Since carbon has an emissivity less than unity, the brightness temperature falls below the true sublimation temperature. Through the red glass of an optical pyrometer, the maximum brightness temperature of the positive crater viewed normally is 3820°K.[3] This is an average value arrived at from observations at three laboratories: the National Bureau of Standards; the General Electric Company, Nela Park Laboratories; and the National Carbon Company Research Laboratories. It has an uncertainty of about 15° due to the difficulty of extrapolating to this temperature from primary standards. The actual arc brightness temperature is believed to be reproducible to within 10°K.

In arriving at the above-recommended operating conditions for this arc, Chaney, Hamister and Glass conducted a great deal of research into the various factors affecting the brightness of the positive electrode of a pure carbon arc. Since 1878 there had been discussion and controversy concerning the constancy of the brightness of the positive crater. Some investigators[4] reported a constant brightness temperature over wide ranges of current and found it to be independent of the kind of carbon used. Others[5] found a difference in the behavior of graphite and amorphous carbon and reported a difference in temperature with arc current. As an example of the variations encountered, Waidner and Burgess of the National Bureau of Standards found an 80° increase in brightness temperature when doubling the arc current, and reported that graphite gave about a 40° higher brightness temperature than amorphous carbon. On the other hand, Guckel and Kohn found that above a certain current density

[3] Dr. H. T. Wensel, in a discussion of this paper as presented at the Temperature Symposium, pointed out that an adjustment of temperature standards to those of the International Temperature Scale brings the value 3810°K, given by Chaney, Hamister and Glass, up to 3820°K.

[4] See for example: H. Kohn and M. Guckel, Zeits. f. Physik **27**, 5–6, 305 (1924); Voille, J. de phys. et rad. (3) **2**, 545 (1893); O. Lummer, *The Liquefaction of Carbon and Attainment of the Solar Temperature* (Braunschweig, 1914).

[5] See for example: F. Patzelt, Zeits. f. tech. Physik **42**, 66 (1923); C. W. Waidner and G. K. Burgess, Bull. Nat. Bur. Stand. **1**, 109 (1904); Blondel, Proc. Int. Elec. Congress, Chicago, 1893.

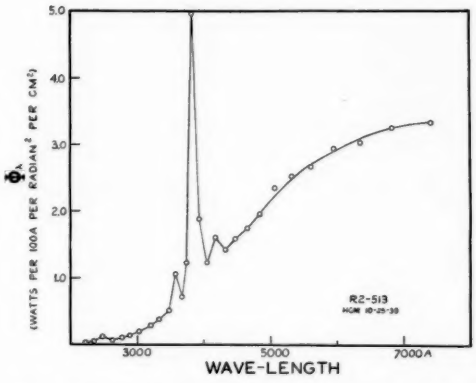

FIG. 1. Spectral energy distribution of the positive crater radiation from a carbon arc employing a $\frac{1}{4}''$ spectroscopically pure graphite positive.

the brightness of the positive crater was constant and the same for all kinds of carbon. In fact, they obtained the same brightness whether the arc operated in air, nitrogen, carbon dioxide, or argon.

Chaney, Hamister and Glass assert that "in the course of a three-year search . . . we succeeded in verifying every statement made on both sides of every important controversy regarding arc behavior." Arc variables were found which reduced the brightness below its maximum value. However, under the prescribed conditions of operation, the same maximum value of the crater brightness was found in both amorphous carbon and graphite. The principal factor affecting the crater brightness was found to be the current density at the active crater area. This has to be greater than a minimum value of about one ampere per mm² to insure the attainment of the maximum crater temperature. Chaney, Hamister and Glass found that this high current density could be reached most easily by use of small graphite positives. Thus the above described arc was concluded to be the most convenient one giving assurance of attaining the maximum brightness temperature. Smaller graphite positives give the maximum temperature readily, but are consumed quickly, while larger graphite positives produce an unsteady arc.

Another method of obtaining a check on the temperature of the positive crater of this arc is to measure its spectral energy distribution, and to match this energy distribution against the energy distribution of blackbodies at different temperatures to find the best fit. If carbon is assumed to be a gray body—that is, to have a constant spectral emissivity—the blackbody temperature that gives the best fit is the true temperature of the arc crater.

Our measurements of spectral energy distribution for this purpose were checked with the large quartz double monochromator at the Nela Park Laboratory of the General Electric Company, through the courtesy of Dr. W. E. Forsythe and Dr. B. T. Barnes. In taking these measurements, a diaphragm with a small hole (2.5 mm in diameter) was placed about 4 cm in front of the arc, so that the radiation measured was limited to that from the active crater area. The energy distribution obtained for the $\frac{1}{4}''$ graphite arc is shown in Fig. 1. The three sharp peaks are due to the cyanogen radiation from the arc stream. However, the rest of the curve, except for a carbon line at 2478A and some general arc stream radiation below 2400A, is due to the continuous spectrum of the carbon crater. Using the portion of the energy distribution curve in the visible region of the spectrum, the color temperature was calculated. This was done by the selected ordinate method[6] and found to be 3990°K.

For matching the energy distribution obtained with that of a blackbody using all wave-lengths regardless of their visibility, it is convenient to plot the energy distribution in another form. Wien's law for the energy distribution of a grey body is

$$\Phi_\lambda = A\,\epsilon_\lambda c_1 \lambda^{-5} e^{-c_2/\lambda T}. \qquad (1)$$

In this equation, Φ_λ is the spectral radiant energy, A is the area of source; ϵ_λ is the spectral emissivity; c_1 is a constant equal to 1.177×10^{-12} watt cm²; λ is the wave-length in cm; c_2 is a second constant equal to 1.432 cm degrees; and T is the temperature. By a simple transformation this equation takes the form:

$$\log(\Phi_\lambda \lambda^5) = \log(A\,\epsilon_\lambda c_1) - \frac{0.6219}{T}\,\frac{1}{\lambda}. \qquad (2)$$

This equation is that of a straight line between

[6] F. T. Bowditch and M. R. Null, J. Opt. Soc. Am. **28**, 500 (1938).

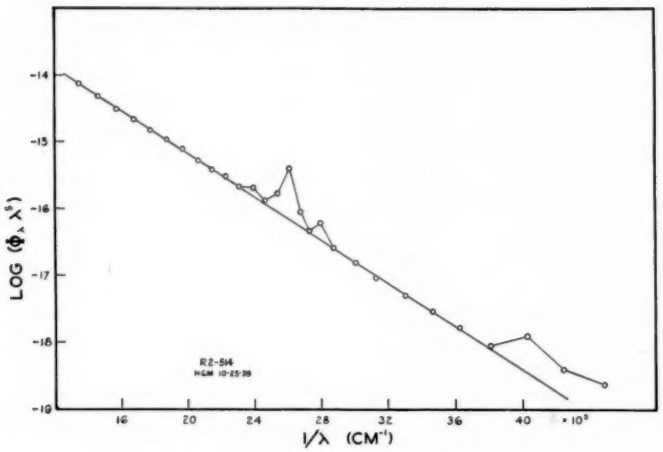

FIG. 2. Logarithmic plot of spectral energy distribution, enabling the arc temperature to be determined from the slope of the line.

the variables log ($\Phi_\lambda \lambda^5$) and $1/\lambda$ for a gray body of constant spectral emissivity. Consequently if we plot the energy distribution for our arc in this form—that is, plot the logarithm of the product of the intensity by the fifth power of the wave-length against the reciprocal of the wave-length—we can expect a straight line if the radiation is similar to that of a blackbody. Furthermore the slope of this straight line will be $-0.6219/T$, so that the temperature of the arc crater can be determined from the slope of the line.

Figure 2 shows the curve obtained for the arc with the $\frac{1}{4}''$ graphite positive when the plot is made in this way. The points fit a straight line very well, except in the region occupied by the cyanogen bands and in the region below 2500A where the arc stream radiation becomes a contributing factor. The fact that the data fit a straight line lends added support to the assumption that the spectral emissivity of carbon is constant over this range of wave-lengths. The slope of the straight line determined by the data corresponds to a temperature of 3860°K. We will call this temperature a "distribution temperature." It may be taken to be the true average temperature of the portion of the positive crater effective in irradiating the monochromator. This will be less than the true maximum temperature, because it includes some lower temperature radiation.

A value for the total emissivity of the graphite can be obtained by measuring the total radiation energy of the arc coming through the diaphragm used in obtaining the spectral energy distribution. The ratio of the total energy found to that expected from a blackbody at the temperature of the arc is the total emissivity of the carbon. Under the assumption that carbon is a gray body this is also the spectral emissivity. The total energy radiated to one side of a blackbody is, by the Stefan-Boltzmann law,

$$\Phi = \sigma A T^4, \tag{3}$$

where Φ is the energy in watts, A and T are the area and temperature of the blackbody, and σ is the Stefan-Boltzmann constant, equal to 5.735×10^{-12} watt/cm^2 degree^4. The energy emitted by the arc crater, diaphragmed as above, in a direction normal to its surface was measured by a calibrated thermopile. This energy was then corrected by subtracting an appropriate amount for the cyanogen band radiation. The ratio of this measured energy to that calculated from Eq. (3) using a temperature of 3860°K for the arc crater was determined as the emissivity. The emissivity calculated in this way is 0.77. A determination of the emissivity of carbon on another arc reported later gives an emissivity of 0.78. Consequently we will take the emissivity of carbon at the arc temperature to be about 0.775.

With the emissivity of carbon determined in this way, the true maximum temperature of the positive crater can be determined from pyrometric observations on the brightest portion of the crater. The brightness temperature of the brightest portion of the crater is 3820°K for the wave-length 6530A. The true temperature is determined from this by the relation

$$\frac{1}{T}-\frac{1}{S_\lambda}=\frac{\lambda \log \epsilon_\lambda}{0.4343c_2}, \qquad (4)$$

where T is the true temperature, S_λ the brightness temperature, and ϵ_λ the spectral emissivity for the wave-length λ. Using an emissivity of 0.775 and a brightness temperature at 6530A of 3820°K, the true maximum temperature of the positive crater is calculated to be 3995°K.

Several different temperature measurements have been reported here, and in order to avoid confusion, they will be reviewed and their relationships discussed.

(1) In the first place there is the distribution temperature, a temperature based on the relative spectral distribution of energy from a diaphragmed area of the positive crater. This temperature is calculated from Wien's law considering only the distribution and not the absolute magnitude of the energy. It is the true average temperature of the area of the crater used, provided only that the carbon is assumed to be strictly a gray body.

(2) The color temperature refers to the visual color effect of the radiation from this area of the crater. It, too, depends on the energy distribution for its calculation, but it includes only the visible radiation. Within accidental errors it would be the same as the distribution temperature if it were not for the fact that the cyanogen band radiation from the arc stream is included in its calculation.

(3) The brightness temperature is based on an optical measurement of the absolute magnitude of the radiation per unit area emitted at a wave-length of 6530A. It is the temperature a blackbody would have if it gave off the same radiant energy per unit area at that wave-length. It applies to the brightest portion of the crater, since that is what one sights on, and is lower than the true temperature of the crater by an amount dependent upon the emissivity of carbon at the arc temperature.

(4) The true maximum temperature of the positive crater is determined from the brightness temperature by use of a numerical value for the emissivity of carbon at this temperature. This temperature and the brightness temperature refer to the brightest portion of the crater, while the distribution temperature and color temperature are necessarily average values over an extended area of the crater. The various temperatures and emissivities have been collected in Table I.

Included in this table are values for another arc besides the standard pyrometric arc. This arc had the same arrangement of electrodes, but used a $\frac{5}{16}''$ spectroscopically pure carbon positive at 12.5 amperes, in place of the $\frac{1}{4}''$ graphite positive. The carbon electrode is made from a lampblack base and consequently has a lower heat conductivity than the graphite. For this reason it forms a more steady arc when larger sized positives are used than does graphite. The larger size of this carbon permits the use of a 0.15″ diameter crater shield in place of the 0.10″ crater shield used for the $\frac{1}{4}''$ positive, so that greater energy is available. Larger sizes than $\frac{5}{16}''$ carbon positives are not recommended, however, in order to avoid wandering of the anode spot.

It will be noticed from the table that the temperatures for this arc are lower than for the standard pyrometric arc, indicating that the current density is not high enough to give the true sublimation temperature of carbon. Consequently it cannot replace the standard pyrometric arc in its use as a standard for optical pyrometry. However, this arc should find wide application because of its steadier burning and larger size. In its operation it is quite similar to that of the graphite arc. Since it is a pure

TABLE I.

	Standard Pyrometric Arc $\frac{1}{4}''$ Graphite Positive	$\frac{5}{16}''$ Carbon Positive
Distribution temperature	3860°K	3840°K
Color temperature	3990°K	3855°K
Brightness temperature	3820°K	3680°K
Maximum true temperature	3995°K	3845°K
Emissivity from total energy measurement	0.77	0.78

carbon the energy distribution and brightness should be quite reproducible, although not so much so as for the graphite pyrometric arc.

For applications where reproducibility of intensity is not so important, and where the presence of a few lines in the spectrum is of no concern, ordinary solid arc carbons of commercial quality are convenient to use. They give a quiet steady arc, and provide an economical source of continuous radiation.

On Interreflections

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1. INTRODUCTION

IN 1927, Buckley showed that interreflections of light within an enclosure with perfectly diffusing walls can be expressed naturally and simply in the form of an integral equation. Since that time a number of investigators[1–9] have contributed to the subject.

In all of this work, however, only the most elementary methods have been used and no attempt has been made to utilize the important developments in integral-equation theory made by Hilbert and by Schmidt. The purpose of the present paper is to formulate the interreflection problem in a precise manner and to indicate the practical conclusions that can be drawn from the more modern theory of integral equations. A few examples are also given, showing how the Hilbert-Schmidt method can be applied to specific cases. Photometric notation is used; but the equations apply equally well to radiant energy, the only change being the substitution of radiosity[10] (watt m^{-2}) in place of luminosity (lumen m^{-2}).

2. INTEGRAL EQUATION FOR THE GENERAL CASE

In the theory of interreflections, the following restrictions are always made: (a) All surfaces are perfectly diffusing. (b) All surfaces are sufficiently symmetrical so that the position of a point can be specified as a function of a single variable. (c) The radiation is homogeneous. (d) The medium within the enclosure is transparent and nondissipative.

Consider any interreflection problem that is subject to the foregoing restrictions. The reflecting surface may be entirely closed or partly open and may have any distribution of reflection factor $\rho(s)$. The surface may be luminous, either because of the passage of light through it from an outer source or by reflection of light. Denote the initial luminosity (with no multiple reflections) by L_0. Then the actual luminosity at a point s, obtained when multiple reflections are present, is

$$L(s)=L_0(s)+\rho(s)\int_S dF, \qquad (1)$$

where dE=illumination produced at s by an infinitesimal area at t. The integration is performed over the entire surface visible from the point s, and thus the integral represents the total illumination at s produced by the entire luminous surface S.

[1] Z. Yamauti, "The light flux distribution of a system of interreflecting surfaces," J. Opt. Soc. Am. **13**, 561 (1926); Res. El. Lab., Tokyo, No. 190, 1927. General formulation of the interreflection problem. Final equation is identical with Fredholm solution of integral equation, but this fact does not seem to be realized and no integral equation is considered.

[2] H. Buckley, "On the radiation from the inside of a circular cylinder," Phil. Mag. **4**, 753 (1927). First statement of integral equations applied to interreflections. Considers the semi-finite circular cylinder. Approximates the kernel by a one-term and a two-term exponential and obtains solution for L_0=const.

[3] H. Buckley, "On the radiation from the inside of a circular cylinder, II," Phil. Mag. **6**, 447 (1928). Solves the finite circular cylinder by using one-term and two-term exponential approximations to the kernel. Considers semi-infinite and finite cylinders illuminated by uniform sky. Obtains exact solution for finite cylinder with longitudinal slot and L_0=const.

[4] H. C. Hottel and F. Keller, "Effect of reradiation on heat transmission in furnaces and through openings," A.S.M.E., Presented at Annual meeting, December, 1932. Numerical Solutions for finite cylinders with $\rho=1$.

[5] Z. Yamauti, "Recherche d'un radiateur intégral au moyen d'un corps cylindrique," Com. Int. Poids et Mesures, Procès-verbaux **16**, 243 (1933). The finite cylinder, with and without circular disk at one end. Uses three-term approximation to the kernel.

[6] H. Buckley, "On the radiation from the inside of a circular cylinder, III," Phil. Mag. **17**, 576 (1934). Considers the finite cylinder with one end closed by a disk. Uses one-term approximation to the kernel.

[7] Z. Yamauti, "On the blackness of cylindrical blackbody," Res. El. Lab., Tokyo, No. 378, 1934. Summary of numerical calculations on finite cylinders, with and without circular disk. Comparison of 1, 2, and 3-term approximations to kernel.

[8] Z. Yamauti, "Etude analytique des "interréflections" dans un cylindre de longeur infinie," Comptes rendus Congrès Int. des Applications de l'Eclairage, Paris, 1937, p. 144. R.G.E. **42**, 293 (1937). Exact solution for portions of circular cylinder of infinite length and L_0=const. Approximate results for other infinite cylinders. Applications to architectural lighting.

[9] W. R. Hedeman, Jr., "The numerical solution of integral equations on the cinema integraph," Sc.D. Thesis, M.I.T., 1939, pp. 151–200. The Cinema Integraph is used in obtaining numerical solutions for a finite circular cylinder.

[10] P. Moon, *Scientific Basis of Illuminating Engineering* (New York, 1936), p. 560.

Since dE is itself a function of the unknown luminosity, Eq. (1) is intrinsically an *integral equation;* and it is now necessary to consider how the equation can best be thrown into the customary Fredholm form:

$$\phi(s) = f(s) + \lambda \int_a^b K(s,t)\phi(t)dt. \tag{2}$$

Evidently, $L(s)$ can be identified with the unknown function $\phi(s)$, and $L_0(s)$ can be identified with the given function $f(s)$. Also, dE may be written

$$\begin{aligned} dE &= E_1(s,t) \cdot L(t)d\sigma \\ &= E_1(s,t) \cdot L(t) \cdot F(t)dt, \end{aligned} \tag{3}$$

where $E_1(s,t)$ = illumination produced at s by a source of unit luminosity and unit area at t, and $d\sigma$ = element of surface at t. According to Postulate (b), $d\sigma$ can be expressed as a function of t, or

$$d\sigma = F(t)dt. \tag{4}$$

Eq. (1) then becomes

$$L(s) = L_0(s) + \int_a^b \rho(s)E_1(s,t)L(t)F(t)dt$$

or

$$L(s) = L_0(s) + \lambda \int_a^b K(s,t)L(t)dt, \tag{1a}$$

where the kernel contains $E_1(s,t)$, $F(t)$, and $\rho(s)$, except perhaps for a constant coefficient which may be identified with λ.

If the reflecting surface is a smooth one having a uniform reflection factor, the kernel will be a continuous function of s and t, though this function may not possess a derivative at all points. One of the theoretical advantages of integral equations over differential equations is that no trouble is introduced by functions that are not analytic. In fact, the theorems of integral-equation theory apply equally well to a much broader class of functions—functions of class L^2. Thus both $L(t)$ and $K(s,t)$ may be discontinuous functions, provided they are of integrable square.

Because of the foregoing property of integral equations, Eq. (1a) is not restricted to reflecting surfaces consisting of a single smooth sheet. The surface may be composed of any number of flat or curved pieces with a different reflection factor on each; and Eq. (1a) will continue to apply, though the kernel will generally have a discontinuity at each break in the surface. For example, the kernel can be written very easily for the interreflections within a rectangular prism of infinite length, having a different reflection factor on each side, dt being an infinitesimal length on the surface. Here $\rho(s)$ is a step-function which must be combined with $E_1(s,t)$ to form the discontinuous kernel. A solution of Eq. (1a) would then give a piecewise-continuous function of four parts, representing the actual luminosity distribution on the four interior faces. The problem is formulated easily, though it has not yet been solved.

Another innovation consists in a broadening of the concept of *reflection factor*. Previous work has restricted reflection factor to positive real values less than unity, but it is instructive to allow this quantity to assume values equal to or greater than unity. The objection may be raised that such values have no physical significance and that any physical reasoning based upon such a concept is therefore invalid. However, surfaces having reflection factors above 100 percent can be constructed by anyone who cares to go to the trouble. An opal glass surface, for instance, can have numerous lamps behind it, these lamps being controlled by small photo-cells on the front surface. By proper arrangements, the total luminosity of the front surface can be made directly proportional to its illumination, or

$$L = \rho E$$

where ρ can be equal to or greater than unity. Such surfaces are probably of no practical value, but the extension of the ordinary concept of reflection factor has advantages, as will be shown in Section 4.

In the Hilbert theory,[11] attention is focused on the *homogeneous* equation

$$\phi(s) = \lambda \int_a^b K(s,t)\phi(t)dt \tag{5}$$

associated with the inhomogeneous equation (2). Ordinarily, Eq. (5) has no solution except the trivial one:

$$\phi(s) \equiv 0.$$

[11] See, for instance, G. Wiarda, *Integralgleichungen* (Leipzig, 1930).

But for certain specific values of $\lambda(\lambda=\lambda_1, \lambda_2 \cdots)$ the equation has solutions:

$$\phi=\phi_1, \phi_2, \cdots.$$

These solutions are called *characteristic functions* or *eigenfunctions* and the corresponding values of λ are called *characteristic values* or *eigenvalues*. Once these functions are found, the solution of the inhomogeneous equation for any arbitrary initial luminosity $L_0(s)$ is a simple matter.

3. Further Restrictions

In setting up the integral equation for a specific interreflection problem, there is considerable leeway in the choice of λ and dt. The factor λ may be taken equal to ρ at any specific value of s, or any numerical value that happens to occur in the kernel may be taken outside the integral sign and incorporated in the λ. The variable t may be taken as an area, a distance, an angle, etc. However, there are advantages to be gained by setting up the problem in the following standardized manner.

In the first place, it is always possible to formulate the integral equation so that dt is equal to an area $d\sigma$.

In the more general type of problem, where the reflection factor is a function of s, the kernel will now be the product of $\rho(s)$ and $E_1(s,t)$ and will usually be asymmetric:

$$K(s,t) \neq K(t,s).$$

Continuous variations in ρ are so unusual in practice that they might well be ignored; but $\rho(s)$ is often a step-function (as in the walls and ceiling of a room), in which case the general formulation of Section 2 must be used. However, if the further restriction be made that $\rho(s)=\rho$ = const, some rather important conclusions can be drawn (Section 4).

In the remainder of the paper, therefore, two additional postulates are used:

(e) $F(t)=1$ in Eq. (4),

(f) $\rho(s)=\rho=\lambda$.

By (f), the parameter λ has a definite physical significance: It is the reflection factor of the surface. The kernel also takes on additional definiteness, since it now is equal to the *illumination produced at s by unit area and unit luminosity at t*. Furthermore, the kernel under these conditions is always symmetric:

$$K(s,t)=K(t,s),$$

which is of theoretical advantage.

4. Theorems

Consider the interreflection problem, subject to restrictions (a), (b), (c), (d), (e), and (f). Under the conditions, a number of general theorems, applying to interreflection problems with all shapes of surface, can be formulated. Some of these statements are of a rather trivial nature and several can be found in treatises on integral equations. Nevertheless, it is felt that a definite statement of the results, both old and new, will be helpful in the further development of interreflection theory. The theorems are as follows: (1) The kernel is always real, positive, symmetric, and bounded $[K(s,t)<1]$. (2) All eigenvalues are real. (3) For a given kernel and given limits, there is one and only one physically realizable eigenvalue (though there may be other "mathematical" or "fictitious" eigenvalues). (4) The physically realizable eigenvalue is always unity for a complete enclosure and is always greater than unity for an incomplete enclosure. (5) For a given kernel and given limits, there is one and only one physically realizable eigenfunction, which is always real, positive, and bounded. (6) The physically realizable eigenfunction for any complete enclosure is a constant. (7) If the kernel is a constant, K_0, there is one and only one eigenfunction, which is a constant. The one and only eigenvalue is

$$\lambda_1=\frac{1}{(b-a)K_0}.$$

(8) The solution of the inhomogeneous equation is

$$L(s)=L_0(s)+\rho\sum_{\nu=1}^{\infty}\frac{\gamma_\nu}{\lambda_\nu-\rho}\phi_\nu(s), \qquad (6)$$

where

$$\gamma_\nu=\int_a^b L_0(s)\phi_\nu(s)ds$$

and $\phi_\nu(s)$ are the normalized orthogonal eigenfunctions for the kernel $K(s,t)$. All the eigenfunctions, including the physically realizable one,

must be used in the summation. Eq. (6) gives the one and only solution of the problem, provided $\lambda_r \neq \rho$. (9) If the physically realizable eigenfunction $\phi_1(s)$ is a constant, then the solution of the inhomogeneous equation for $L_0(s) = L_0$ is a constant:

$$L(s) = \frac{\lambda_1 L_0}{\lambda_1 - \rho}.$$

Justification for the foregoing statements will now be given.

(1) The kernel is equal to the illumination produced by a source of unit area and unit luminosity. From physical considerations, this quantity is always real and positive. Moreover, by a well-known law of optics, the illumination can never exceed the luminosity of the source which produces this illumination; so the kernel can never exceed the value unity. It is also evident that an interchange of s and t will not alter the physics of the subject, so the kernel is symmetric.

(2) That all eigenvalues of a real symmetric kernel are real is well known.[12]

(3,4) That every symmetric kernel of class L^2 has at least one eigenvalue is proved in treatises on integral equations. The conclusion that there is no more than one physically realizable eigenvalue is based on physical reasoning. In the homogeneous equation (5), $\phi(s)$ represents the steady-state distribution of luminosity on the surface with no light source present. We may assume that at some remote time in the past a pulse of radiant energy entered the system but no more energy has entered since that time. Eq. (5) then gives the resulting steady-state luminosity distribution on the surface. From physical considerations, one expects that unless the conditions are very unusual the result will be the trivial solution

$$\phi(s) \equiv 0.$$

This is also in accordance with the theory of homogeneous integral equations, which proves that nontrivial solutions exist only for the *eigenvalues* $\lambda_1, \lambda_2 \cdots$.

Imagine that the reflection factor ρ of the walls of a complete enclosure can be continuously varied. If $\rho \ll 1$, any pulse of radiant energy that enters the enclosure will be quickly absorbed by the walls, giving the steady-state solution, $\phi(s) \equiv 0$. As ρ is raised, the number of reflections (required to give a luminosity too small to measure) increases, but eventually the same trivial solution is reached. However, if ρ is exactly unity, there is no absorption of radiant energy and a valid solution is obtained with the eigenvalue $\lambda_1 = 1$ and the eigenfunction $\phi_1(s)$ found by solution of Eq. (5).

If $\rho > 1$, the walls add some radiant energy at each reflection. Thus the total radiant energy within the enclosure increases continuously without limit, $\phi(s) \to \infty$, and no solution of (5) exists. If the enclosing surface contains openings, similar reasoning leads to the belief that again there is only one eigenvalue but this value is always greater than unity to compensate for the loss of energy through the openings.

(5) According to integral-equation theory, one or more eigenfunctions are associated with each eigenvalue. In an interreflection problem, therefore, there must be one or more physically realizable eigenfunctions associated with the one physically realizable eigenvalue. These eigenfunctions represent the steady-state luminosities produced by a flash of radiant energy. Obviously, they will be real, positive and bounded.

That there is not more than one physically realizable eigenfunction can be shown as follows. Consider a definite surface with a specified reflection factor. The only possible cause of differences in the steady-state luminosity distribution on this surface would be a difference in the intensity and duration of the initiating flash. But the only effect of these variables is to introduce a multiplying factor, which would disappear in the normalizing process. Thus, the existence of two physically realizable eigenfunctions, associated with λ_1, would mean that identical conditions led to different results, a conclusion that would violate a fundamental postulate of physics.

An alternative proof that there is not more than one physically realizable eigenfunction and not more than one physically realizable eigenvalue is based on the orthogonality of the eigenfunctions. It is well known that if any two eigenfunctions $\phi_i(s)$ and $\phi_j(s)$, of a symmetrical kernel $K(s,t)$, are associated with two different

[12] Reference 11, p. 34.

eigenvalues λ_i and λ_j, then $\phi_i(s)$ and $\phi_j(s)$ are orthogonal:

$$\int_a^b \phi_i(s)\phi_j(s)ds = 0. \tag{7}$$

Suppose that $\phi_i(s)$ is a physically realizable eigenfunction with eigenvalue λ_i. Then it is positive throughout the range $a \leqq s \leqq b$, since a negative luminosity has no meaning. Any other eigenfunction $\phi_j(s)$ associated with eigenvalue $\lambda_j (j \neq i)$ must be positive over part of the interval ab and negative over part of this interval, in order that Eq. (7) shall be satisfied. Thus any eigenfunction $\phi_j(s)$, associated with the eigenvalue $\lambda_j (j \neq i)$, is not physically realizable.

(6) This theorem was first stated by Whitmore[13] and is easily proved by use of the "solid-angle method."[14] Given a complete enclosure of arbitrary shape. Assume that the steady-state luminosity of the walls is a constant, $\phi(s) = C \neq 0$. Then at any point P on the surface (with the possible exception of isolated points and lines), radiant energy is being received through a solid angle of 2π, and thus the resulting illumination is

$$E_p = \phi(s) = C.$$

But

$$E_p = \int_S dE = \int K(s,t)\phi(t)dt.$$

Therefore, $\phi(s) = C$ is a solution of the homogeneous equation with $\lambda_1 = 1$; or $\phi_1(s) = C$ is an eigenfunction. There is no other physically realizable eigenfunction. (Theorem 5.)

(7) If $K(s,t) = K_0$, Eq. (5) becomes

$$\phi(s) = \lambda K_0 \int_a^b \phi(t)dt.$$

The right side of the equation is independent of s, so the one and only eigenfunction is

$$\phi_1(s) = \text{const} = k.$$

Substitution gives

$$\lambda_1 = \frac{1}{(b-a)K_0},$$

which is the one and only eigenvalue.

(8) Proved in treatises on integral equations.

(9) Let $\phi_1(s) = \text{const} = k$ be the physically realizable eigenfunction. Also, $L_0(s) = L_0 = \text{const}$. The orthogonality condition is

$$\int_a^b \phi_1(s)\phi_\nu(s)ds = 0 \quad (\nu = 2, 3, 4, \cdots)$$

or

$$\int_a^b \phi_\nu(s)ds = 0.$$

But by Theorem 8,

$$\gamma_\nu = \int_a^b L_0(s)\phi_\nu(s)ds = L_0 \int_a^b \phi_\nu(s)ds = 0$$

for all eigenfunctions except $\phi_1(s)$. For the physically realizable eigenfunction,

$$\gamma_1 = L_0 K \int_a^b ds = L_0 k(b-a)$$

and normalization of $\phi_1(s)$ gives

$$k^2 = 1/(b-a).$$

Thus the solution is

$$L(s) = L_0 + \frac{\rho\gamma_1 k}{\lambda_1 - \rho} = \frac{\lambda_1 L_0}{\lambda_1 - \rho}.$$

5. Kernels

Kernels for various surface configurations can be obtained by the usual methods of illumination calculation.[15] A few simple kernels are given in Table I. In each case, the conditions of Section 3 are used; that is, $\rho = \lambda = \text{const.}$, and dt is taken as an element of area. When the element is infinite in length, unit length can be considered, so dt is numerically equal to arc length.

6. Simple Examples

The interreflection problem for the *sphere* can be solved by elementary methods, without the use of integral equations. It may be of interest, however, to apply the Hilbert-Schmidt method to obtain the well-known results for interreflections in a sphere.

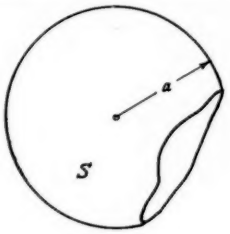
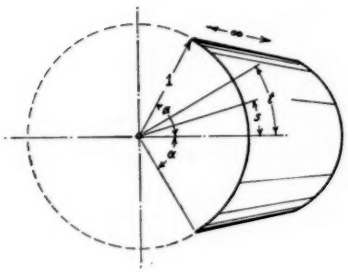
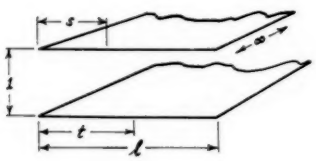
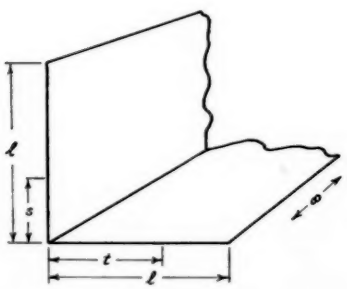
A unique property of the sphere is that the illumination produced by an element $d\sigma$ of spherical surface is the *same at all points* of the

[13] W. H. Whitmore, "Interreflections inside an infinite cylinder," J. Math. Phys. **17**, 218 (1939). Solution is given for infinite cylinder with *arbitrary initial luminosity*.

[14] P. Moon, "Basic principles in illumination calculations," J. Opt. Soc. Am. **29**, 108 (1939).

[15] Reference 10, Chap. X.

TABLE I. *Kernels.*

FORM	KERNEL
1. Sphere 	$\frac{1}{4\pi a^2}$
2. Circular cylinder of infinite length 	$\frac{1}{4}\sin\left\lvert\frac{t-s}{2}\right\rvert$
3. Parallel plates 	$\frac{1}{2}[1+(t-s)^2]^{-\frac{1}{2}}$
4. Perpendicular plates 	$\frac{1}{2}\frac{st}{[s^2+t^2]^{\frac{1}{2}}}$
5. Plates at any angle α	$\frac{1}{2}\frac{st\sin^2\alpha}{[s^2+t^2-2st\cos\alpha]^{\frac{1}{2}}}$

sphere wall. For unit luminosity, this illumination has the value

$$E=\frac{1}{4\pi a^2}d\sigma,$$

where a is the radius of the sphere. In applying integral equations to the problem, it has been customary[16,17] to take $d\sigma$ as the area of an elementary ring and to express everything in terms of an angle θ. Such a procedure destroys the innate simplicity of the problem and gives a

[16] H. Buckley, "Some problems of interreflection," Proc. C.I.E., p. 888, 1928. Considers sphere, semi-infinite cylinder, finite cylinder, infinite cylinder with slot, and two semi-infinite planes.

[17] Reference 10, p. 328.

TABLE. I. *(Continued.)*

FORM	KERNEL
6. Circular cylinder, finite length	$\frac{1}{2}\left\{1-\frac{\lvert t-s\rvert[(t-s)^2+6]}{[(t-s)^2+4]^{\frac{1}{2}}}\right\}$
7. Rectangular prism, finite length*	$\frac{1}{\pi(1+x)}\left\{\ln\left[\frac{x^2+(s-t)^2}{(s-t)^2}\right]-\ln\left[\frac{1+x^2+(s-t)^2}{1+(s-t)^2}\right]+x\left[\frac{1}{[1+(s-t)^2]^{\frac{1}{2}}}\tan^{-1}\frac{x}{(1+(s-t)^2)^{\frac{1}{2}}}+\frac{x}{[x^2+(s-t)^2]^{\frac{1}{2}}}\tan^{-1}\frac{1}{(x^2+(s-t)^2)^{\frac{1}{2}}}\right]\right\}$
8. Square prism, finite length*	$\frac{1}{\pi}\left\{\frac{1}{[1+(s-t)^2]^{\frac{1}{2}}}\tan^{-1}\frac{1}{(1+(s-t)^2)^{\frac{1}{2}}}+\frac{1}{2}\ln\left[\frac{1+(s-t)^2}{(s-t)^2}\right]-\frac{1}{2}\ln\left[\frac{2+(s-t)^2}{1+(s-t)^2}\right]\right\}$
9. Parallel circular disks	$\frac{1+s+t}{[(1+s+t)^2-4st]^{\frac{1}{2}}}$

* Approximate, since the illumination at s is considered to be independent of position around the rectangle $ABCD$ and to have the arithmetic mean of the actual values around this rectangle.

kernel which is not only a function of θ but is also asymmetric. A better plan is not to specify the form of the elementary area $d\sigma$ but to take this quantity directly as

$$d\sigma = dt.$$

The kernel is then a constant $(1/4\pi a^2)$, and the integral equation is

$$L(s) = L_0(s) + \rho\int_0^S \left[\frac{1}{4\pi a^2}\right] L(t)dt, \qquad (8)$$

where S is the total area of the portion of the sphere that is being employed, irrespective of the shape of this surface.

The homogeneous equation is

$$\varphi(s) = \lambda\int_0^S \left[\frac{1}{4\pi a^2}\right]\varphi(t)dt. \qquad (8a)$$

By theorem 7, the one and only eigenfunction is

$$\varphi_1 = \text{const.}$$

and the one and only eigenvalue is

$$\lambda_1=\frac{4\pi a^2}{S}. \tag{9}$$

For a complete sphere, $S=4\pi a^2$ and Eq. (9) shows that $\lambda_1=1$. Thus, with $L_0=0$, a steady-state solution (other than $\varphi(s)=0$) can be obtained only if the reflection factor is $\rho=\lambda_1=1.00$. The resulting luminosity $\varphi_1(s)$ is the same all over the sphere. For an incomplete sphere, a nontrivial solution of Eq. (8a) is obtained only for ρ greater than unity and having the value found from Eq. (9). For example, with a hemisphere, $\lambda_1=2$; for a lune having the angle $\pi/4$, $\lambda_1=8$, etc. It is interesting to note that the luminosity remains uniform [$\varphi_1(s)=$const.] over whatever portion of the sphere is employed, a phenomenon that does not take place in the corresponding problem of the cylinder of infinite length (Section 7 of this paper). The eigenfunction is normalized by making

$$\int_0^S \varphi_1^2(s)ds=1$$

or

$$\varphi_1=\frac{1}{S^{\frac{1}{2}}}. \tag{10}$$

The inhomogeneous equation (8) can now be solved for any initial distribution $L_0(s)$. According to Theorem 8,

$$L(s)=L_0(s)+\frac{\rho}{(\lambda_1-\rho)S}\int_0^S L_0(s)ds$$

$$=L_0(s)+\frac{\rho(L_0)_{Av}}{\lambda_1-\rho}, \tag{11}$$

where $(L_0)_{Av}$=average initial luminosity of the surface S. In the special case of $L_0(s)=L_0=$const, Eq. (11) becomes

$$L(s)=\frac{\lambda_1 L_0}{\lambda_1-\rho}, \tag{11a}$$

where λ_1 is obtained from Eq. (9). For a complete sphere with $L_0(s)=$const, Eq. (11a) reduces to the well-known formula,

$$L(s)=\frac{L_0}{1-\rho}. \tag{11b}$$

It is interesting to note that Eq. (11b) is not confined to spheres but *applies equally well to any complete enclosure* with $L_0=$const. By Theorem 6, the physically realizable eigengunction for any complete enclosure is

$$\phi_1(s)=k$$

and by Theorem 4,

$$\lambda_1=1.$$

According to Theorem 9, the solution of the inhomogeneous equation for $L_0(s)=L_0$ is a constant:

$$L(s)=\frac{\lambda_1 L_0}{\lambda_1-\rho}=\frac{L_0}{1-\rho}. \tag{11b}$$

Since a solution of the inhomogeneous equation is unique, Eq. (11b) is the one and only luminosity distribution for a uniform initial distribution, $L_0(s)=$const, provided $\rho\neq 1$.

If $L_0(s)\neq$const, the conclusions are not so satisfactory. By Theorems 3 to 6, there is but one physically realizable eigenvalue:

$$\lambda_1=1$$

and but one physically realizable eigenfunction:

$$\phi_1(s)=k.$$

Normalizing

$$k^2\int_s d\sigma=1, \quad \text{or} \quad k^2=1/S,$$

where S is the area of the complete enclosure. If there were no other eigenfunctions, the solution of the inhomogeneous equation would be identical with that for the sphere, Eq. (11). Generally the fictitious eigenfunctions will destroy this simplicity and will give a solution that is a function of s.

For *infinite parallel* plates, the homogeneous equation is

$$\varphi(s)=\lambda\int_{-\infty}^{\infty}\tfrac{1}{2}[1+(t-s)^2]^{-\frac{1}{2}}\varphi(t)dt. \tag{12}$$

Assume the solution, $\varphi(s)=C$. Then Eq. (12) becomes

$$1=\frac{\lambda}{2}\int_{-\infty}^{\infty}[1+(t-s)^2]^{-\frac{1}{2}}dt=\frac{\lambda}{2}\int_{-\infty}^{\infty}[1+x^2]^{-\frac{1}{2}}dx=\lambda.$$

Thus the physically realizable eigenfunction

TABLE II. *Eigenvalues for the infinite cylinder.*

α(RADIANS)	k_1	λ_1
0.2	11.97	144.3
0.4	5.971	36.65
$\pi/6$	4.509	21.33
0.6	3.915	16.33
$\pi/4$	2.945	9.673
0.8	2.887	9.335
1.0	2.258	6.099
$\pi/3$	2.143	5.592
1.2	1.829	4.345
1.4	1.512	3.286
$\pi/2$	1.299	2.687
1.6	1.266	2.603
1.8	1.066	2.136
2.0	0.8978	1.8060
2.5	0.5541	1.3070
3.0	0.2214	1.0490
π	0.0000	1.0000

is a constant and $\lambda_1=1$, in accordance with Theorem 4.

Another simple case is that of *two infinite plates intersecting at any angle* α. The homogeneous equation is

$$\varphi(s)=\lambda\int_0^\infty \frac{1}{2}\frac{st\sin^2\alpha}{[t^2+s^2-2st\cos\alpha]^{\frac{1}{2}}}\varphi(t)dt. \quad (13)$$

Assume $\varphi(s)=C$. Then

$$1=\frac{\lambda s\sin^2\alpha}{2}\int_0^\infty \frac{t\,dt}{[t^2+s^2-2st\cos\alpha]^{\frac{1}{2}}}$$
$$=\frac{\lambda}{2}(1+\cos\alpha).$$

Thus the physically realizable eigenfunction is a constant, and the associated eigenvalue is

$$\lambda_1=\frac{2}{1+\cos\alpha}. \quad (14)$$

7. Circular Cylinder of Infinite Length

An exact solution will be obtained for the interreflections in a strip cut from a circular cylinder of infinite length. The angular width of the strip is 2α, and the integral equation is

$$L(s)=L_0(s)+\rho\int_{-\alpha}^{\alpha}\left[\frac{1}{4}\sin\left|\frac{s-t}{2}\right|\right]L(t)dt$$
$$(0\leqq\alpha\leqq\pi). \quad (15)$$

The related homogeneous equation is

$$\varphi(s)=\lambda\int_{-\alpha}^{\alpha}\left[\frac{1}{4}\sin\left|\frac{s-t}{2}\right|\right]\varphi(t)dt. \quad (15a)$$

Assume the solution:

$$\varphi(s)=A\cosh\frac{ks}{2} \quad (16)$$

where A and k are constants that are to be evaluated.

Substitution of Eq. (16) in Eq. (15a) gives

$$\cosh\frac{ks}{2}=\frac{\lambda}{k^2+1}\left\{\cosh\frac{ks}{2}+k\cos\frac{s}{2}\cdot\left[\sin\frac{\alpha}{2}\sinh\frac{k\alpha}{2}-\frac{1}{k}\cos\frac{\alpha}{2}\cosh\frac{k\alpha}{2}\right]\right\}.$$

Since $\cos(ks/2)$ and $\cosh(ks/2)$ are linearly independent, their respective coefficients can be equated to zero, giving

$$\left.\begin{aligned} k_\nu&=(\lambda_\nu-1)^{\frac{1}{2}},\\ \sin\frac{\alpha}{2}\sinh\frac{k_\nu\alpha}{2}&=\frac{1}{k_\nu}\cos\frac{\alpha}{2}\cosh\frac{k_\nu\alpha}{2}.\end{aligned}\right\} \quad (17)$$

Thus the assumed form of solution, Eq. (16), is correct; but nontrivial solutions are obtained only if λ has one of the values λ_ν given by Eq. (17).

If $\lambda_\nu>1$, Eq. (17) may be written

$$By=\coth y, \quad (18)$$

where

$$B=\frac{2}{\alpha}\tan\frac{\alpha}{2},\quad y=\frac{\alpha}{2}(\lambda_\nu-1)^{\frac{1}{2}}.$$

Evidently, for a given value of α there is a unique value of λ_1, determined by the intersection of the straight line By and the hyperbolic function $\coth y$. A few such values are given in Table II. Eq. (18) determines the one and only physically realizable eigenvalue (Theorem 3) for a given portion of a circular cylinder of infinite length.

With this eigenvalue is associated an eigenfunction:

$$\varphi_1(s)=A_1\cosh\frac{k_1 s}{2}, \quad (16a)$$

where $k_1=(\lambda_1-1)^{\frac{1}{2}}$. Eq. (16a) is the unique

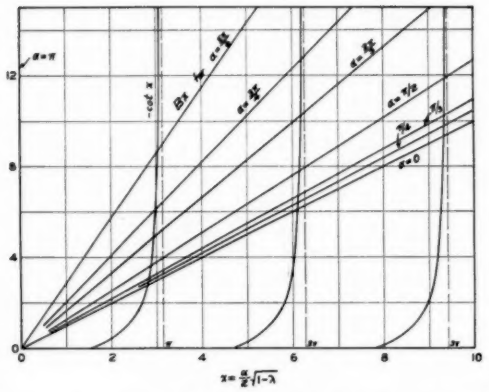


FIG. 1. Curves for the determination of eigenvalues λ_ν ($\nu=2, 3, 4\cdots$) for the circular cylinder of infinite length.

solution for the interreflection problem with $L_0(s)=0$, and indicates that there is no nontrivial steady-state solution of the homogeneous equation unless the reflection factor of the cylinder wall is exactly $\rho=\lambda_1$, when the luminosity $L(s)=\varphi_1(s)$ is a hyperbolic cosine function. The function can be normalized:

$$A_1^2\int_{-\alpha}^{\alpha}\cosh^2\left(\frac{k_1 s}{2}\right)ds=1$$

or

$$A_1^2=\left[\alpha+\frac{1}{k_1}\sinh k_1\alpha\right]^{-1}. \tag{19}$$

The foregoing completes the solution of the homogeneous equation. For the inhomogeneous equation, however, the other eigenvalues must be considered. If $\lambda_\nu<1$, Eq. (17) becomes

$$Bx=-\cot x, \tag{20}$$

where

$$B=\frac{2}{\alpha}\tan\frac{\alpha}{2},\quad x=\frac{\alpha}{2}(1-\lambda_\nu)^{\frac{1}{2}}.$$

The eigenvalues are determined by the intersections of a straight line and the cotangent curve, Fig. 1. The corresponding eigenfunctions are

$$\varphi_\nu(s)=A_\nu\cosh\left[\frac{s}{2}(\lambda_\nu-1)^{\frac{1}{2}}\right]$$

$$=A_\nu\cos\left[\frac{s}{2}(1-\lambda_\nu)^{\frac{1}{2}}\right]. \tag{16b}$$

It is easily shown that the φ's form an orthogonal set. If normalized,

$$A_\nu^2=\left[\alpha+\frac{1}{k_\nu}\sin k_\nu\alpha\right]^{-1}, \tag{21}$$

where $k_\nu=(1-\lambda_\nu)^{\frac{1}{2}}$, $\nu=2, 3, 4, \cdots$. The complete set of eigenfunctions contains also sinh and sine terms, which must be used if $L_0(s)$ is not an even function. The eigenvalues and coefficients for these terms are evaluated exactly as were the even functions considered in the previous paragraphs and will not be considered here.

The solution of the inhomogeneous equation, for an arbitrary initial luminosity $L_0(s)$ and for any reflection factor ρ, is now easily obtained. According to Theorem 8, the solution is

$$L(s)=L_0(s)+\rho\sum_{\nu=1}^{\infty}\frac{\gamma_\nu}{\lambda_\nu-\rho}\varphi_\nu(s), \tag{6}$$

where

$$\gamma_\nu=\int_{-\alpha}^{\alpha}L_0(s)\varphi_\nu(s)ds\quad(\nu=1, 2, 3, \cdots).$$

For the special case of $L_0(s)=L_0=\text{const}$,

$$\gamma_1/L_0=A_1\int_{-\alpha}^{\alpha}\cosh\frac{k_1 s}{2}ds=\frac{4A_1}{k_1}\sinh\frac{k_1\alpha}{2},$$

$$\gamma_\nu/L_0=A_\nu\int_{-\alpha}^{\alpha}\cos\frac{k_\nu s}{2}ds=\frac{4A_\nu}{k_\nu}\sin\frac{k_\nu\alpha}{2},$$

$$(\nu=2, 3, \cdots).$$

Substitution in Eq. (6) gives

$$L(s)=L_0\left\{1+4\rho\left[\frac{M_1}{\lambda_1-\rho}\cosh\frac{k_1 s}{2}+\sum_{\nu=2}^{\infty}\frac{M_\nu}{\lambda_\nu-\rho}\cos\frac{k_\nu s}{2}\right]\right\}, \tag{6a}$$

where

$$M_1=\frac{\sinh(k_1\alpha/2)}{[k_1\alpha+\sinh k_1\alpha]}, \tag{22}$$

$$M_\nu=\frac{\sin(k_\nu\alpha/2)}{[k_\nu\alpha+\sin k_\nu\alpha]}\quad(\nu=2, 3, \cdots). \tag{23}$$

The case of greatest practical significance is probably with $\alpha=\pi/2$. This semicircular cylinder with $L_0(s)=L_0$ represents, for instance, long rooms with vaulted ceilings and dark walls,

tunnels, long ceiling coffers, semicircular reflectors with matt surfaces. For the cosh-term of Eq. (6a),

$$k_1 = 1.299,$$
$$\lambda_1 = 2.687,$$
$$M_1 = 0.2072.$$

The corresponding values for $\nu = 2, 3, \cdots$ are obtained by use of Eqs. (20) and (23) and are given in Table III. The fifth and sixth columns of the table give the values of the terms in Eq. (6a) for two extreme reflection factors, $\rho = 1.00$ and $\rho = 0.10$. As ν increases, the terms decrease in magnitude in a very satisfactory manner, so that an accurate sum is easily obtained. The sum is the value of $L(s)/L_0$ for $s = 0$, which is the place of minimum luminosity. Thus at the darkest part of a semicircular cylinder having $\rho = 1.00$, interreflections raise the luminosity by 47.7 percent; while near the edges, interreflections are even more effective. A set of luminosity curves for various reflection factors is given in Fig. 2, the values having been computed by the use of Eq. (6a).

Table III indicates that the effect of all the cosine-terms is small. In the worst case considered in the table, an error of 1.0 percent is introduced by neglecting the cosine terms while with $\rho = 0.10$ the worst error is 0.2 percent. A study of the equation for other values of α gave errors which in no case exceeded 2 percent. Therefore, for engineering purposes it is generally satisfactory to write Eq. (6) in the simplified form:

$$L(s) \cong L_0(s) + \frac{\rho N}{\lambda_1 - \rho} \cosh \frac{k_1 s}{2}, \qquad (6b)$$

where

$$N = \frac{k_1 \int_{-\alpha}^{\alpha} L_0(s) \cosh \frac{k_1 s}{2} ds}{[k_1 \alpha + \sinh k_1 \alpha]}. \qquad (24)$$

TABLE III.

ν	λ_ν	k_ν	M_ν	TERMS IN EQ. (6a), $s=0$ FOR $\rho=1.00$	FOR $\rho=0.10$
0				1.0000	1.0000
1	+ 2.687	1.299	+0.2072	+0.4914	+0.03204
2	− 12.40	3.661	+0.05025	−0.0150	−0.0017
3	− 60.48	7.841	−0.01048	+0.000682	+0.000069
4	−140.6	11.90	+0.00452	−0.000128	−0.000013
5	−253.0	15.94	−0.00252	+0.000040	+0.000004
			$L(s)/L_0 =$	1.4770	1.0304

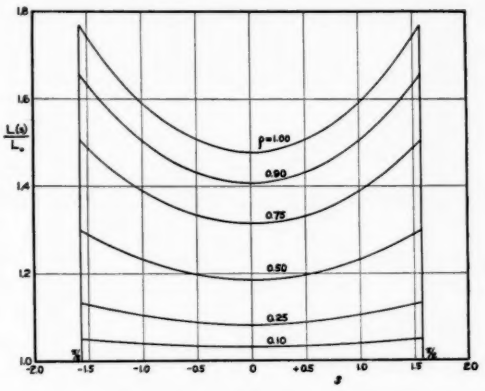


FIG. 2. Luminosity distribution in a semicircular cylinder of infinite length.

Eq. (6b) is a very simple solution of the interreflection problem for a strip of any width cut from a circular cylinder. It applies to any reflection factor and to any even function $L_0(s)$. The values of k_1 and λ_1 are obtained from Table I. If the initial luminosity distribution is not an even function, an additional term containing the hyperbolic sine must be used.

In the special case of $L_0(s) = L_0 = \text{const}$, Eq. (6b) reduces to

$$L(s) \cong L_0 \left[1 + \frac{4\rho M_1}{\lambda_1 - \rho} \cosh \frac{k_1 s}{2} \right], \qquad (6c)$$

where M_1 is obtained from Eq. (22).

8. SUMMARY

In the foregoing paper, the aim has been to formulate the interreflection problem in a precise manner and to indicate how the Hilbert-Schmidt theory of integral equations can be applied to the problem. A number of examples are given, including a new solution for a strip cut from a circular cylinder of infinite length. The solution is given in the form of an infinite series; but if all terms but the first be ignored, the result will ordinarily be correct to within one or two percent.
In conclusion, I wish to thank Professor Philip Franklin for his helpful suggestions on the solution of integral equations.

Experiments on the Thermoluminescence of Some Common and Unusual Minerals

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Introduction

ALTHOUGH interest in the ultraviolet fluorescence and phosphorescence of minerals has become widespread in recent years, other common forms of luminescence have received much less attention. This is especially true of thermoluminescence, the property of substances through which they emit visible light when heated below ordinary redness. This phenomenon has been known for a long time (a variety of fluorite having been named "chlorophane" because of its green thermoluminescence) but was often not distinguished as a separate property. Thus Dana's *A Textbook of Mineralogy* (third edition, 1922) gives several examples under the heading, "phosphorescence," mentioning that light is also produced by crushing certain minerals with a hammer. The latter is now classified as triboluminescence, or the emission of light by solid bodies when fractured. No heat is necessary for its production. Both types of luminescence are relatively faint, being best observed in the dark.

The thermoluminescence of minerals has been investigated by Haberlandt,[1] Köhler and Leitmeier,[2] Brown, Gunnell[3] and others. Kranck[4] described the effect of particle size and rate of heating, and Wick *et al.*[5–7] of previous exposure to high voltage cathode rays, and of x-rays on both minerals and synthetic preparations. Various experiments were made by Cohn *et al.*[8–10] on glass. Wick[11] has noted that previous exposure to radium emanation intensifies the thermoluminescence of many minerals, while Przibram[12,13] and his associates reported a similar effect on triboluminescence. A theoretical discussion of the mechanism is given by Muto.[14] The foregoing authors, and others indicated later, have also studied the phenomenon in various distinct species or classes of minerals by means of the spectrograph. However, much of the pertinent literature consists of casual and insufficient descriptions included in papers on some other subject.

The purpose of this paper is to note some of the qualitative aspects of thermoluminescence, which is of a complex nature, to correlate this property if possible with other forms of luminescence, and to assemble a list of readily available minerals for the convenience of future investigators. Since specimens from foreign localities have been used in nearly all previous work on this subject, it was decided to investigate primarily the thermoluminescence of American minerals and such others as are to be found generally in mineral collections.

[1] H. Haberlandt, "Luminescence investigation of fluorites and other minerals, (III)," Mitt. d. Inst. f. Radiumforschung, No. 391, 1–10, 1937.

[2] A. Köhler and H. Leitmeier, "The natural thermoluminescence of minerals and rocks," Zeits. f. Krist. **87**, 146–80 (1934). (Includes a bibliography of 34 references extending back to 1664.)

[3] W. L. Brown, "Photophosphorescence in minerals," Univ. of Toronto Studies, Geol. Series, No. 35, 19–35, 1933. E. M. Gunnell, "Phosphorescence of minerals. Part 2—Being a discussion of thermo-luminescence in minerals," Rocks and Minerals **3**, 12–14 (1928).

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[6] F. G. Wick, "Effect of x-rays on thermoluminescence," J. Opt. Soc. Am. **14**, 33–44 (1927).

[7] F. G. Wick and M. K. Slattery, "Thermoluminescence excited by x-rays. Further experiments upon synthetically prepared materials," J. Opt. Soc. Am. **16**, 398–408 (1928).

[8] B. E. Cohn, "The absorption spectra of glasses which exhibit thermoluminescence," J. Col.-Wyo. Acad. Sci. **1**, No. 3, 33 (1931).

[9] B. E. Cohn and W. D. Harkins, "Thermoluminescence in glasses which contain two activators," J. Am. Chem. Soc. **52**, 5146–54 (1930).

[10] R. E. Nyswander and B. E. Cohn, "Measurements of thermoluminescence of glass exposed to light," J. Opt. Soc. Am. **20**, 131–6 (1930).

[11] F. G. Wick, "An experimental study of the triboluminescence of certain natural crystals and synthetically prepared materials," J. Opt. Soc. Am. **27**, 275–85 (1937).

[12] K. Przibram, "On the discoloration of compressed rocksalt," Mitt. d. Inst. f. Radiumforschung, No. 196, 43 (1927); K. Przibram and E. Kara-Michailova, "On radioluminescence and radio-photoluminescence," *Ibid.* No. 149, 511 (1922); F. Urbach, "Concerning the luminescence of the alkali halogenides," *Ibid.* No. 261, 353; No. 262, 363 (1930).

[13] H. Haberlandt and K. Przibram, "Concerning the fluorescence of fluorite," Mitt. d. Inst. f. Radiumforschung. No. 313, 235 (1933).

[14] T. Muto, "The theory of thermoluminescence in some crystals," Sci. Papers Inst. Phys. Chem. Research (Tokyo), **28**, 207–20 (1936).

Discussion and Methods

Thermoluminescence was tested for by heating a small specimen in the dark, either on an electric hot plate, or in a large spoon over a nonluminous gas flame. A 30-ml Pyrex beaker was inverted over the specimen to prevent loss by decrepitation.

The specimen was heated at a moderate rate, and the color and intensity of thermoluminescence observed as the temperature rose. In all cases, the luminescence disappeared at a temperature in the neighborhood of redness. However, if heating was discontinued and the specimen allowed to cool before this extinction point was reached, the luminescence could be made to reappear on heating again. In some minerals, such as the Franklin fluorite and the hackmanite from Ontario, this process could be repeated a number of times before the thermoluminescence was exhausted.

With few exceptions, the minerals tested emitted light of only one color, usually preceded by a dim white glow at low temperature when the thermoluminescence first became visible. A similar white glow could also be seen in a few minerals at high temperatures, though more often the principal thermoluminescence color was faintly visible right up to extinction.

That the white glow is probably produced by a different mechanism from the colored luminescence was shown by heating a composited specimen of red Franklin fluorite for three hours in an oven at 155°C, after which the characteristic brilliant green could not be obtained, but only a dim white at intermediate and high temperatures. An unheated control showed the normal thermoluminescence for this mineral. Also, the Deming, New Mexico, fluorite which goes through a series of color changes when the temperature is raised, failed to show any of these on cooling from just below the extinction point. It then emitted only a pale bluish white which persisted down to a relatively low temperature.

It should also be noted that the natural color, if any, of the fluorites tested was destroyed by heating to the extinction point, or slightly beyond. Exposure to sunlight for a week, or to strong ultraviolet light for five minutes, failed to regenerate the thermoluminescence of Franklin fluorite after it had been heated over a Meker burner for two hours.

The minerals were also tested for fluorescence, phosphorescence and triboluminescence. The latter is produced only by fracturing a substance, and should not be confused with ordinary friction effects. It can be demonstrated by grinding the specimen in a mortar, or scratching with a file in darkness. If the mineral is triboluminescent, scratching will leave a momentary luminous track in it, whereas ordinary friction sparks tend to fly out from the surface, and are seldom produced by minerals softer than steel.

It was found that the fluorites and a specimen of pectolite could be heated above the extinction point for thermoluminescence without noticeably diminishing the intensity of triboluminescence. Thus it seems probable that there is a fundamental difference in the means by which the two forms of luminescence are produced. This test was not made on the other triboluminescent minerals listed below.

The triboluminescence of some of the fluorites and the above specimen of pectolite was greatly, but not completely, diminished by grinding under water. This bears out Wick's[11] statement that triboluminescence is in part due to an electric discharge in air, as shown by the presence of nitrogen bands in its spectrum.

A further effect of grinding was observed with the Deming, N. M., fluorite which was thus caused to regain part of its thermoluminescence after this had been completely destroyed by heating over a Meker burner for 15 minutes. The regenerated luminescence was a faint white which came up at an intermediate temperature and persisted until the first appearance of redness of the hot plate, becoming blue just before extinction. Five minutes irradiation with the long waves of an argon bulb before reheating neither intensified nor diminished it. An unground control portion of the same previously heated sample showed no thermoluminescence whatever. This phenomenon was called "tribothermoluminescence" by Nyswander and Cohn[15] who first reported it.

Fluorescence and phosphorescence were tested for by means of an argon bulb, an iron spark, and

[15] R. E. Nyswander and B. E. Cohn, "Tribothermoluminescence," Phys. Rev., **36**, 1257–1260 (1930).

TABLE I. *Fluorite.*

TYPE NO.	SPEC. NO.	LOCALITY	NATURAL COLOR	THERMOLUM.	TRIBOLUM.	FLUOR. (ARGON BULB)	PHOS. (ARGON BULB)
I	1	Franklin, N. J.	Rose red	Brilliant green	White	None	None
II	2	Mt. Adam, N. Y.	Colorless	Strong bluish green; deep orange yellow; very faint blue	White faintly bluish	Light blue	None
II	3	Mt. Adam, N. Y.	Dark purple	Strong greenish white; fine blue green; strong blue-yellowish white; very faint blue	Faintly bluish white	None	None
II	4	Wilberforce, Ont.	Black (Antozonite)	Greenish white; fine blue-green; strong yellowish white; very faint blue green	Very faint white	None	None
II	5	Prescott, Arizona	Bluish green	Strong greenish yellow; purple white; deep orange yellow	Strong bluish white	None	None
II	6	Prescott, Arizona	Dark purple	Faint bluish white; green; strong greenish yellow; purple white; deep orange yellow; faint bluish	Strong yellowish white	None	None
III	7	Cumberland, Eng.	Colorless	Deep cobalt blue	White	Deep cobalt blue	None
III	8	Cumberland, Eng.	Dark green	Deep cobalt blue	White	Deep cobalt blue	None
III	9	Silesia, Prussia	Black (Antozonite)	Deep blue	—	None	None
IIIA	10	Ferry Co., Washington	Pale green	Bright blue-yellow-white; deep cobalt blue	Faint white	Faint blue	None
IIIA	11	California	Very pale green	Bright blue yellow-white; deep cobalt blue	Bluish white	None	None
IV	12	France	Pale green	Yellow; light blue	White	Faint blue	None
V	13	Ouray, Colorado	Colorless	Yellow-blue-white; light blue; deep yellow	Very faint white	Very pale blue	None
V	14	In limestone near Mt. Adam, N. Y.	Purple	Yellow-blue-white; deep blue; faint yellow	Faint white	None	None
VI	15	Deming, New Mexico	Bluish green	Yellow-blue-white; deep blue; orange yellow; strong cobalt blue	White	Deep cobalt blue	None
VII	16	Clay Center, Ohio	Brown	None	White	Yellowish green	Greenish white
—	17	(Yttrofluorite) Hundholmen, Norway	Greyish white	Pale yellow; bluish white	None	None	None
—	18	(Fluocerite) Finbo, Sweden	Greyish	Dim white	White	None	None

an R. & M. lamp, with and without a Corning No. 986 dark red purple filter, depending on which source of ultraviolet light gave the best results with the mineral in question.

The variation in heat stability of fluorescence, thermoluminescence and triboluminescence seems to indicate a fundamentally different *modus operandi* for each. Thus, the fluorites from

Mt. Adam, N. Y. and Deming, N. M. lost only a part of their fluorescence intensity when heated just to extinction of thermoluminescence. Further heating of the second over a Meker burner for 15 minutes completely destroyed the fluorescence but caused only a very slight decrease in triboluminescence. The fluorescence of cancrinite from Ontario, wernerite from Quebec, and calcium larsenite from Franklin, was also somewhat decreased by heating to complete destruction of thermoluminescence, or slightly beyond, as was the phosphorescence of South Dakota calcite. Heating to redness almost completely removed the phosphorescence of calcium larsenite, but caused no visible change in either the fluorescence or phosphorescence of Franklin willemite.

Thermoluminescence of Fluorite

The fluorites, which exhibit more varied thermoluminescence than any other class of minerals, will be discussed first. In the following table, the colors listed are those obtained by steadily increasing the temperature, a semicolon denoting a perceptible change in hue. The initial faint glow is not mentioned unless noticeably different from the next succeeding color, or unusually persistent. In some cases, several distinct colors were observed at once, as though tiny spots on the specimen were emitting different ones, or a sort of oscillatory emission was taking place, instead of a single blended hue being produced (Table I).

Arsandaux,[16] Haberlandt[1] and others, have described the thermoluminescence of fluorite qualitatively, while Steinmetz and Gisser,[17] Haberlandt,[1,28] Iimori,[18] Wick,[11] and Iwase[19–23] have studied the thermoluminescence spectra of various fluorites, the last two also having investigated the effect of x-rays in intensifying thermoluminescence and restoring it after previous destruction by heating. The consensus is that thermoluminescence in this mineral usually depends on the presence of traces of various rare earth elements.

The above specimens can be conveniently divided into several types on the basis of the colors emitted, as follows:

Type I. Strong green (variety: "Chlorophane"). —Wick[11] found, by spectrographic examination of the thermoluminescence of a similar fluorite from Franklin, that the activating impurity was Mn. Under certain conditions, faint lines of Sm and Dy also appeared. Haberlandt[1] and Arsandaux[16] reached a like conclusion on examining this type of fluorite from various foreign localities. The above specimen (No. 1) contained enough Mn to give a faint borax bead test. A semi-quantitative spectrographic analysis showed considerable Mn and about 0.4 percent Mg, 0.005 percent Pb, 0.002 percent Sn, and 0.4 percent Fe. It would seem that the latter impurities are not involved to any extent in the thermoluminescence. Unfortunately the concentration of rare earth elements in such calcium minerals is too low to be readily detected spectrographically by direct examination, so that it is not known whether these are present in specimen No. 1.

Type II. Bluish or yellowish green at moderate temperatures plus a high temperature yellow.—The thermoluminescence of this group is more complex in origin than the preceding one. Steinmetz and Gisser,[17] Iwase,[23] Wick[7] and others have noted thermoluminescence of more than one color in a single specimen of fluorite. While the low temperature green of this type may be due in part to Mn (which could not be found by a borax bead test in specimen No. 2; nor by spectrographic analysis in No. 5 and No. 6, although small amounts of Mg, Fe, and Si were present), it is more probably due to certain rare earth impurities. Thus Wick[11] found with synthetic CaF_2 preparations containing traces of various

[16] H. Arsandaux, "Violet fluorspars," Bull. Soc. franc. mineral **58**, 268–77 (1935).

[17] H. Steinmetz and A. Gisser, "The spectrum of the thermoluminescence of fluorite," Naturwiss. **24**, 172 (1936).

[18] S. Iimori, "The constitution of phosphorescence centers in fluorite," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **20**, 189–200 (1933).

[19] E. Iwase, "Thermoluminescence spectra of fluorites (I); Thermoluminescence spectra of fluorites from Obira," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **22**, No. 461, 233–41 (1933).

[20] E. Iwase, "Thermoluminescence spectra of fluorites (II); Division of the bands in the thermoluminescence spectrum of fluorites into two groups," Sci. Papers Inst. Phys. Chem. Research (Tokyo), **23**, 22–23 (1933).

[21] E. Iwase, "Thermoluminescence spectra of fluorites (III); thermoluminescence spectra of fluorites from Wilberforce," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **23**, 153–61 (1934).

[22] E. Iwase, "Thermoluminescence spectra of fluorites (IV); Spectra of thermoluminescence restored by x-rays," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **23**, 212–23 (1934).

[23] E. Iwase, "Bands in the thermoluminescence spectrum of fluorite from Obira, Japan," Nature **131**, 909–10 (1933).

TABLE II. *Calcite.*

TYPE NO.	SPEC. NO.	LOCALITY	NATURAL COLOR	THERMOLUM.	TRIBOLUM.	FLUOR. (ARGON BULB)	PHOS. (ARGON BULB)
I	1	Weehawken, N. J.	Transluc. white	Faint white; deep orange yellow	None	Salmon red (R. & M.+filter)	Red flash (Fe-spark)
I	2	Franklin, N. J.	Transpar. greyish	Strong white; med. yellowish white	Faint white	Strong red (Fe-spark)	Red flash (Fe-spark)
I	3	Franklin, N. J.	Opaque white	Dim white	None	None	Very faint flash (Fe-spark)
I	4	Franklin, N. J.	Granular white	Good yellow	None	Very pale pink (Fe-spark)	Red flash (Fe-spark)
I	5	Franklin, N. J.	Granular white	Dim yellowish white	None	Pink (Fe-spark)	Red flash (Fe-spark)
I	6	Franklin, N. J.	Transpar. greyish	Pale yellow	None	Strong red (Fe-spark)	Red flash (Fe-spark)
I	7	Gardin, Okla.	Pale yellow	Strong white; deep orange yellow	White	None	None
I	8	Unknown	Opaque white	Dim yellowish white	Faint white	None	None
I	9	Haverstraw, N. Y.	Light yellow	Pale yellow	None	Yellowish green	White
I	10	Imperial Co., Calif.	Transluc. grey	Deep orange yellow	None	Bright orange red	None
I	11	Portland, Conn.	Greyish	Deep orange yellow	None	None	None
I	12	Joplin, Mo.	Colorless	Deep orange yellow	None	None	None
I	13	French Creek, Pa.	White	Faint yellow	None	None	None
I	14	Chimney Rock, N. J.	Pale yellow	Deep yellow	None	None	None
I	15	Easton, Pa.	White columnar	Strong deep orange; fading; strong yellow	Faint white	None	None

rare earths, that after exposure to radium, Tb caused a green thermoluminescence and Sm an orange. A strong yellow band of Dy was also found in the thermoluminescence spectrum of certain fluorites before irradiation.

With the specimens from Mt. Adam, the luminescence of No. 3 which was closely associated with allanite, started much sooner than that of No. 2, and was also more brilliant at higher temperatures. It seems probable that this intensification can be traced either to the weak radioactivity of the allanite, or to the concentration of rare earth elements in its neighborhood. The former fact may also account for the dark purple color. The similarity of behavior of specimens 3 and 4 is worthy of note, No. 4 being a true antozonite, having been so decomposed by the radioactivity of an associated mineral that it gives off a strong odor of ozone when fractured. Neither antozonite No. 4 nor No. 9 is very strikingly different from an ordinary fluorite of the same type, possibly because the known intensification of thermoluminescence and triboluminescence by radioactivity tends to decrease again with long continued exposure. Although nothing is known of the associations of specimens

TABLE II—*Continued*

TYPE NO.	SPEC. NO.	LOCALITY	NATURAL COLOR	THERMOLUM.	TRIBOLUM.	FLUOR. (ARGON BULB)	PHOS. (ARGON BULB)
I	16	Summit, N. J.	Greyish	Deep orange yellow	None	None	None
I	17	Fanny Gouge Mine, N. C.	White	Deep orange yellow	None	None	None
I	18	Unknown	Pink	Deep orange yellow	None	None	Faint blue (Fe-spark)
I	19	Brewster Co., Texas	Pale pink	Faint dark yellow	None	Very pale yellow (blue Fe-spark)	Strong bluish white (red flash Fe-spark then strong blue)
I	20	Brewster Co., Texas	Colorless	Fainter than No. 19	None	None (faint bluish Fe-spark)	Very dim white (red flash Fe-spark then faint bluish white)
I	21	Brewster Co., Texas	Pale blue green	Deep yellow	None	None (blue Fe-spark)	Like No. 19 under both argon bulb and Fe-spark
II	22	South Dakota	White	Pale bluish white	None	Faint yellow green	Strong bluish white
II	23	Paterson, N. J.	Colorless	Faint bluish white	None	None	Extremely faint white
II	24	Franklin, N. J.	Deep orange	Very faint white	None	None	None
II	25	California	Reddish travertine	Faint white	None	None	None
III	26	Ogdensburg, N. J.	Yellow-brown	None	None	Pale greenish yellow	Strong yellowish green; white
III	27	Cincinnati, Ohio	Greyish crust on gravel	None	None	Strong green	Strong yellowish green; strong white

5 and 6, the fact that the thermoluminescence of No. 6 appears long before that of No. 5 and persists to a higher temperature, coupled with the very dark purple color of the former, suggests that, as with the Mt. Adam fluorite, No. 6 was associated with some radioactive mineral while No. 5 was not.

All except specimen No. 5 in this group show a very faint high temperature bluish glow somewhat similar to the initial thermoluminescence. This color could not be accurately observed because of brilliant redness of the plate at that point. Neither the initial nor the highest temperature color is particularly like the triboluminescence color to the eye.

Type III. Pure cobalt blue, deepening as the temperature rises.—This is strikingly similar to the fluorescence of some of the specimens (7 and 8) showing it. Haberlandt, Karlik and Przibram,[24] Wick,[11] and Haberlandt and Przibram[13] have found by spectrographic examination of blue fluorescence in fluorites, that its main cause is a trace of Eu. It seems probable that the same element is also involved in this type of thermoluminescence.

Type IV. Low temperature yellow with a higher temperature blue.—Specimen 12 above was found spectrographically to contain a trace of Mn. Since this is much less than the amount in specimen 1 and the former has no green thermoluminescence, it does not seem that Mn is of importance in this case. A trace of Pb was also found. The faint fluorescence of specimen 12 (and

[24] H. Haberlandt, B. Karlik and K. Przibram, "The fluorescence of the fluorites (II)," Mitt. d. Inst. f. Radiumforschung, No. 336, 151–61 (1934).

13 in the next group) is similar in color to the higher temperature blue thermoluminescence, possibly indicating a common source for the two. However, such fluorescence is lacking in specimen 14.

Type V. Similar to Type IV plus a high temperature yellow.

Type VI. Similar to Type V plus a very high temperature blue.—A faint trace of Mn and a spectrographic trace of Pb were found in specimen 15, but as with specimen 12, these do not appear to be involved to any extent in the complex thermoluminescence. The blue fluorescence is here similar in color to two distinct portions of the thermoluminescence.

Specimen 16 which shows fluorescence, phosphorescence and triboluminescence is unusual in not being thermoluminescent. The fluorescence and phosphorescence, which are somewhat similar to that of curtisite, amber and certain bituminous compounds, are probably due to inclusions of that nature. The mineral quickly blackens when heated. Haberlandt[25] describes brown fluorites from Kelly Island, New York, St. Louis, Missouri, and Riedlingen in Baden, all having a similar yellowish green fluorescence due to bituminous inclusions. No doubt the second is actually from the same locality as specimen 16 above.

As far as the authors know, the above noted thermoluminescence of fluocerite and yttrofluorite has not been previously reported.

Among other halides cryolite (Greenland), colorless and red sylvite, colorless halite, and dark red carnallite (all from Carlsbad, New Mexico), were not thermoluminescent. The colorless halite was moderately triboluminescent, and a blue halite showed very faint thermoluminescence.

Thermoluminescence of Calcite

The frequency with which calcite shows yellow thermoluminescence and the comparative absence of triboluminescence are notable. Apparently there is no simple relation between fluorescence and thermoluminescence, either of which may exist without the other. A similar conclusion was also reached by Iwase[26] in a study of Japanese calcites. (Table II.) Spectrographic studies of the thermoluminescence of calcite by Iimori,[27] Iwase,[26] and Haberlandt[1, 28] indicate that traces of rare earths generally serve as the activating impurity; although the first author pointed out a similarity with the spectrum obtained by the combustion of calcium. Hata[29] found by analysis of 10 Japanese calcites that the strongest thermoluminescence occurred in those having about 0.05 percent of rare earth oxides, chiefly Pr and Nd, and that the ratio of (Fe, Mn, Zn) to rare earth oxides was smallest in such specimens. Thus it appears that traces of the rare earths are fully as common in calcite as in fluorite. None of the above authors found any sign of bands due to Mn in the thermoluminescence spectrum of calcite, which is somewhat surprising as this element is a common impurity and is known to activate other forms of luminescence. However, Nichols, Howes and Wilber[30] have shown that Mn bands in the cathodoluminescence spectrum of CaO+Mn preparations are completely displaced by the lines of certain rare earths, if the latter are added in amount comparable to the Mn. Similarly, rare earths might repress Mn bands in a thermoluminescence spectrum, which would tend to indicate that conclusive proof of the inactivity of Mn has not yet been furnished. Furthermore, Wick[11] found that a synthetic $CaCO_3$ preparation containing 1 percent of $MnCO_3$ showed a weak orange thermoluminescence after exposure to radium. Unfortunately, the purity of the $CaCO_3$ is not mentioned, and as no ordinary treatment would remove traces of rare earths, if present, these may also have contributed to the thermoluminescence.

[25] H. Haberlandt, "Luminescence investigations of fluorites and other minerals (I)," Mitt. d. Inst. f. Radiumforschung, No. 350, 591 (1934).

[26] E. Iwase, "Luminescence of calcite (II); Thermoluminescence spectra of Japanese calcites," Bull. Chem. Soc. Japan, **11**, 523–28 (1936).

[27] S. Iimori, "Thermoluminescence spectrum of calcite," Sci. Papers Inst. Phys. Chem. Research (Tokyo), **20**, 274–84 (1933).

[28] H. Haberlandt, "Luminescence investigations of fluorites and other minerals (II)," Mitt. d. Inst. f. Radiumforschung, No. 374, 663–6 (1935).

[29] S. Hata, "The minor constituents of thermoluminescent calcite," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **20**, 163–9 (1933).

[30] E. L. Nichols, H. L. Howes and D. T. Wilber, "Cathodoluminescence and the luminescence of incandescent solids," Carnegie Institution of Washington, Publication No. 384, 172 (1928).

Brown[31] showed that the intensity of reddish fluorescence in calcite rises sharply with increase in Mn content to a maximum at 3.5 percent and then decreases more slowly to zero at 17 percent. In preliminary tests, a fluorescent sample from Franklin, and another, but much weaker, one from Weehawken, New Jersey, showed thermoluminescence in the reverse order. This suggested that a Mn content high enough for maximum fluorescence tends to inhibit yellow thermoluminescence, while a lower percent of Mn (as in specimen No. 1) either aids in its production, or at least does not interfere with the effect of rare earth impurities. In other words, this type of thermoluminescence was thought to be inversely proportional to reddish fluorescence. To check this, specimens 1, 2, and 3 were selected on the basis of the relative intensities of their fluorescence and thermoluminescence, it being assumed that a very high Mn content was the cause of the comparative lack of both in 3. The fluorescence of 1 is similar in color to that of 2, but so much fainter that nearly all visible light must be filtered out in order to observe it. With the iron spark, all three show a very fugitive red phosphorescence which seems to be typical of manganiferous calcites, that of No. 3 being faintest. All three were found by a semi-quantitative spectrographic analysis to contain practically the same amount of Mn, estimated at 2–3 percent, showing that there is no simple connection between Mn content and thermoluminescence intensity. In these cases, therefore, other factors must also influence fluorescence under unfiltered ultra violet light. A number of additional impurities were found to be present in the following approximate percentages:

Specimen No.	Mg	Pb	Sn	Fe
1	0.3	0.005	0.001	0.1
2	0.4	0.20	0.0025	0.04
3	0.4	0.02	0.001	0.40

It will be noted that the strongly fluorescent specimen 2 contains more Pb and less Fe than the others, while the strongly thermoluminescent specimen 1 is very low in Pb and intermediate in iron. Although, this may have a bearing on the relative intensity of these forms of luminescence, it seems better justified to interpret the above simply as another illustration of the extreme complexity of the problem.

As a further check, three more specimens (numbers 4, 5 and 6) showing a gradation in fluorescence under the iron spark, were tested for thermoluminescence with the result that 4 and 6 were in agreement with the hypothesis while 5 was not.

In the course of these tests, it was noted that the thermoluminescence of 2 which is also triboluminescent, came up much sooner than that of the standard specimen (No. 1) which is not. This suggested a connection between triboluminescence and an unusually brilliant initial thermoluminescence. Although specimens 7 and 15 also thermoluminesced at a lower temperature than number 1, number 8 did not, again showing the difficulty of making any simple generalization.

Specimen 15 not only gave the brightest thermoluminescence of any calcite tested, but also showed a distinct change from reddish orange to yellow, separated by an interval of perceptible fading (not observed in any other specimen) when heated at a uniform rate. The mineral appears to be secondary, having replaced a vein of cross-fiber asbestos in a serpentine which carries traces of thorianite and its alteration products. A semi-quantitative spectrographic analysis revealed a number of impurities in the following approximate percentages:—Mg—1.0 percent, Mn—0.3 percent, Fe—0.01 percent, Pb—0.005 percent, and Sn—0.005 percent.

In the case of calcite having a strong bluish white phosphorescence under the argon bulb (specimens 19, 21, 22) several factors also seem to act independently. Thus, 19 and 21, which were found spectrographically to contain a small amount of Mn, show the usual yellow thermoluminescence, whereas that of 22 is precisely similar in appearance to the phosphorescence. Specimens 19 and 21 resemble 1 and 3 in not having reddish Mn fluorescence under the iron spark, and in showing a momentary reddish phosphorescence[32] which quickly gives way to the

[31] W. L. Brown, "Fluorescence of manganiferous calcites," Univ. of Toronto Studies, Geol. Series, No. 36, 45–53 (1934).

[32] O. I. Lee, "A new property of matter: reversible photosensitivity in hackmanite from Bancroft, Ontario," Am. Mineral. **21**, 764–76 (1936); "Further study of a new property of matter: reversible photosensitivity in hackmanite from Bancroft, Ontario," Proc. Opt. Soc. Am., Spring Meeting, Corning, N. Y., March 4–6, 1937.

TABLE III. *Minerals with thermoluminescence similar to calcite. Type 1.*

SPEC. NO.	MINERAL	LOCALITY	NATURAL COLOR	THERMOLUMINESCENCE	TRIBO-LUMINESCENCE	FLUO-RESCENCE (ARGON	PHOS-PHORESCENCE BULB)
1	Mangan-apatite	Portland, Conn.	Dark green	Yellow	Faint white	Pale yellow (R. & M.	None + filter)
2	Fluor-apatite	Hai Chow, China	Dark brown	Greenish yellow	—	—	—
3	Apatite	Nelson Co., Virginia	Pale yellow	Faint dark orange yellow	—	—	—
4	Apatite	McAfee, N. J.	Grey green	Strong orange yellow	—	—	—
5	Wernerite	Point au Chêne, Que.	Light yellow	Bright yellow; greenish yellow	—	Strong yellow	—
6	Wernerite	Bolton, Mass.	Reddish lavender	Strong orange-yellow	—	—	—
7	Wernerite	Easton, Pa.	Purple	Faint dark yellow	—	—	—
8	Scapolite	Near Mt. Adam, N. Y.	Pale yellow-ish white	Yellowish white	—	—	—
9	Spodumene	Portland, Conn. and Newry, Maine	Pale pinkish white	Strong orange-yellow then strong light yellow	—	—	—
10	Cancrinite?	Bancroft, Ontario	White	Strong greenish yellow	—	Pale salmon	Pale bluish white

more persistent blue. Specimen 20 on the other hand, contains a similar amount of Mn, but is much weaker in both phosphorescence and thermoluminescence, indicating either a deficiency of other activating agents or an excess of inhibitors. Again, specimen 23 which shows a pale bluish white thermoluminescence somewhat like that of 22 is not fluorescent and practically lacking in phosphorescence.

Specimens 26 and 27 are unusual in that they are both strongly fluorescent and phosphorescent, but not thermoluminescent, constituting a third type.

The following other carbonate minerals were found to give a dim white thermoluminescence: strontianite, Hamm, Westphalia; witherite, Hexham, Northumberland, England; thaumasite, Paterson, New Jersey. The first also showed a pale greenish fluorescence and a very strong greenish white phosphorescence, while the second was weakly phosphorescent. All three appeared substantially free of calcite. A number of carbonates were found to be nonthermoluminescent as follows:

Aragonite (acicular)	Hoboken, New Jersey and Allentown, Pennsylvania
Magnesite	Hoboken, New Jersey and San Jose, California
Hydromagnesite	Hoboken, New Jersey
Dolomite	Jefferson City, Tennessee
Ankerite	West Pittston, Pennsylvania
Siderite	West Pittston, Pennsylvania
Rhodocrosite	Franklin, New Jersey
Azurite	Bisbee, Arizona
Spurrite	New Mexico
Cerussite	Idaho
Nesquehonite	Lansford, Pennsylvania

Two others appear to be an exception, namely, twinned aragonite from Roswell, New Mexico, which has an orange yellow thermoluminescence, similar to that of calcite, and smithsonite from Texas which shows a brilliant, but brief red.

Certain calcareous fossils, notably belemnites from the Liassic sediments of Vendée, France and gryphea cymbium from a later horizon in the same region have been reported by Déribéré[33] to show a blue thermoluminescence. Specimens supplied by him were tested and found to give a

[33] M. Déribéré, "Luminescence in Liassic Belemnites of Vendée," Comptes rendus Soc. Geol. France, **44** (1938).

TABLE IV. *Lepidolite type of thermoluminescence.*

SPEC. NO.	MINERAL	LOCALITY	NATURAL COLOR	THERMOLUMINESCENCE	TRIBOL. (CLEAVAGE LUMINESCENCE)	FLUORES.	PHOSPHOR.
1	Lepidolite (book)	Newry, Maine	Pale lilac	Orange red	Strong bluish white	None	None
2	Lepidolite (scales)	Portland, Conn.	Deep lilac	Faint orange red	Faint bluish white	None	None
3	Lepidolite (scales)	Lac du Bonnet, Manitoba	Deep lilac	Fainter and more orange than No. 2	None	None	None
4	Tourmaline (Rubellite)	Portland, Conn.	Transparent red	Very faint orange red	None	None	None

pale bluish white luminescence, thus corresponding to calcite, Type II.

THERMOLUMINESCENCE SIMILAR TO CALCITE, TYPE I

A number of seemingly unrelated minerals also showed pronounced yellow thermoluminescence similar to that of calcite, Type I. That mechanical inclusions of the latter are not the source of this luminescence, was shown by failure to find more than a very faint trace of carbonate in specimens 1, 2, 6 and 9 (below) (Table III). Here, as with the calcites, a uniformity of behavior is noted in the apatites. Specimen 1 contains enough Mn to give a borax bead test, specimen 2, several tenths of a percent of Mn and somewhat less Pb (spectrographically) while nothing is known about the composition of specimens 3 and 4. The greenish tinge in the thermoluminescence of 2 indicates the presence of some active impurity beside the more usual one. The possibility suggests itself that the thermoluminescence of apatite may be caused by small amounts of Mn or rare earths replacing Ca in the (CaF) portion of the molecule. Hoffman[34] in his study of the coloration of apatite found that Mn imparted a green or blue, and Pr and Nd, alone or in combination with Mn, various shades of brown. He mentions that a few specimens had blue or greenish blue thermoluminescence, with fluorescence of various colors. Byler[35] has described the preparation of calcium phosphate phosphors by heating the acid salts to drive off water, causing a distortion of the crystal lattice with the production of luminescent properties. A similar distortion may account in part for the behavior of specimen number 1, which shows some surface alteration. Besides the different activating effect of the various possible metal impurities, a further complication in the behavior of apatite might be introduced by the substitution of carbonate, silicate, sulfate or arsenate for part of the phosphate, and of chlorine or hydroxyl for the fluorine. Whatever the nature of the activating agent responsible for yellow thermoluminescence, it must be of very common occurrence under widely different geochemical conditions. Specimen 1 is from a lithium pegmatite, 3 from a Nelsonite-Ilmenite dike, 4 from a crystalline limestone, and 2, unknown. The similarity of fluorescence and thermoluminescence in specimen 1 is worthy of note.

The following other anhydrous phosphates were found to be nonthermoluminescent:

Lithiophyllite (8 percent Li_2O)	Branchville, Connecticut
Triphylite	Peru, Maine
Beryllonite	Stoneham, Maine
Pyromorphite	Oracle, Pinal Co., Arizona
Monazite	North Carolina

Little can be said about the scapolite group except that the fluorescence and the greenish tinge in the thermoluminescence of 5 may be due to a trace of U. Haberlandt[1] found lines of this element in the fluorescence spectrum of a number of European scapolites. The greenish thermoluminescence of No. 10 is no doubt due to admixed hackmanite. While the identity of this

[34] J. Hoffman, "On the atomic and ionic coloration of artificially prepared and natural apatites," Chem. d. Erde **11**, 552–75 (1938).

[35] W. H. Byler, "Inorganic phosphors without metallic activators," J. Am. Chem. Soc. **60**, 1247–52 (1938).

mineral is uncertain, the predominately yellow thermoluminescence, and the absence of reversible photosensitivity serve to distinguish it from the hackmanite with which it is associated. Judging from the minerals in Tables II and III, triboluminescence is not often associated with simple yellow thermoluminescence.

Lepidolite Type of Thermoluminescence

Specimens of lepidolite and one of rubellite are grouped together because they all show a faint orange red thermoluminescence which appears only at a relatively high temperature (Table IV). A spectrographic examination of the first three specimens showed small amounts of Na, Rb, and Cs in the order of decreasing concentration, besides the expected Li. The relative amounts of each element appeared to be similar, except that Na was a little higher in the one from Portland. K and F were not found, possibly because of lack of sensitivity in the plate used. No conclusions could be drawn as to the cause of the difference in thermoluminescence intensity. Specimen 4 is similar to the lepidolites in that it undoubtedly contains Li. The difference between this type of thermoluminescence and that of the other common lithium minerals (spodumene, Table III, specimen 9; amblygonite, Table VII, specimens 5, 6, 7) is of interest. Apparently, the type of compound is of as much importance as the presence of lithium.

The strong triboluminescence of specimen 1, which is best seen by vigorously passing a knife blade between the cleavage planes, serves to emphasize the dependence of the phenomenon on sudden rupture along such planes. It was somewhat fainter in the muscovite mentioned below,† the high dielectric constant of which would lead to the conclusion that a static discharge is largely responsible. However, the unusual strength of both thermoluminescence and triboluminescence in specimen 1 suggests the influence of an activating impurity. Cleavage-luminescence has been reported in a mica from Kilmer, Quebec by Meen.* Among other micas, a specimen of phlogopite from Franklin was alone in showing a faint white thermoluminescence. The following micas and tourmalines were nonthermoluminescent:

Muscovite†	Bedford, New York†
Biotite	Bedford, New York†
Polylithionite	Kangerdluarsuk, Greenland
Zinnwaldite	Amelia, Virginia
Tourmalines:	
"Newry Green"	Newry, Maine
Brick red	Newry, Maine
Green	Portland, Connecticut
Black	Portland, Connecticut

Thermoluminescence of Feldspars

The thermoluminescence of feldspars is characterized by its relative faintness, that of the sodic ones being usually white, and of the potash feldspars, bluish white. The lack of response to ultraviolet light is notable in both. Iimori *et al.*[36,37] have studied the intensifying effect of x-rays on the thermoluminescence of feldspars, and have reported analytical and spectrographic data. Kranck[4] has investigated the effect of particle size and rate of heating, and Royer[38] the thermoluminescence of orthoclase in crystalline schists (Table V). Specimen No. 1, among the few albites showing a strongly colored thermoluminescence, does not give a borax bead test for Mn although associated with manganapatite. The source of its luminescence, and of the more common white, are both unknown.

The characteristic pale bluish white thermoluminescence of the above potash feldspars (Table VI) is stronger in specimens closely associated with gummite and uraconite (No. 2), uranophane (No. 4), columbite (No. 5), or allanite (No. 6), than in unaltered crystals imbedded in quartz (No. 1) and not associated with such minerals. The blackened material surrounding grains of gummite in 2 gives a brighter and more persistent thermoluminescence than either 4 or 3, the last being farther from the gummite in the same specimen and

* "Cleavage=luminescence in mica," read by V. B. Meen at the 48th annual meeting of the Mineralogical Society of America, Washington, D. C., December 28–30, 1937.

[36] S. Iimori and E. Iwase, "Spectrographic investigation of the thermoluminescence of feldspars," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **28**, 147–51 (1935).

[37] S. Iimori, J. Yoshimura and S. Hata, "Thermoluminescent feldspars occurring in the vicinity of Naegi," Bull. Inst. Phys. Chem. Research (Tokyo) **13**, 1091–3 (1934).

[38] L. Royer, "Thermoluminescence of schistose and eruptive rocks of Algeria," Comptes rendus **204**, 602–4 (1937).

somewhat darkened. It seems likely that this intensification of thermoluminescence in specimen 2 was caused by long exposure to the weak radioactivity of cyrtolite or of the uranium minerals associated with it. It may also be due in part to the concentration of rare earths in the neighborhood of these minerals. In either case, the effect would not be as great in specimen 4 which has only a comparatively recent coating of secondary uranophane and is practically unaltered. The influence of weak radioactivity may also account for the relatively strong thermoluminescence of specimen 5 associated with columbite, and of specimen 6, with allanite. Columbite from the Baylis quarry is no doubt rendered slightly radioactive by the presence of specks and films of a greenish uranium mineral which it generally contains. Iimori, Yoshimura and Hata[39] have reported strong thermolumines-

[39] S. Iimori, J. Yoshimura and S. Hata, "Radioactive minerals from Shinden, Gifu Prefecture," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **23**, 209–11 (1934).

TABLE V. *Soda feldspars.*

SPEC. NO.	MINERAL	LOCALITY	NATURAL COLOR	THERMOLUMINESCENCE	TRIBOLUM.	FLUORES.	PHOSPH.
1	Albite (Clevelandite)	Portland, Conn.	Pale bluish white	Pale green	White	None	None
2	Albite (Clevelandite)	Amelia, Va.	White	Faint bluish green	White	None	None
3	Albite	Newry, Maine	White	Faint yellowish white	None	None	None
4	Albite	Franklin, N. J.	White	Pale bluish white	Very faint white	None	None
5	Albite	Bedford, N. Y.	Colorless (xl.)	Very faint white	None	None	None
6	Albite	North Carolina	White	Faint white	None	None	None
7	Oligoclase	Hawk, N. C.	Colorless	Bluish white	White	None	None
8	Oligoclase	New York City	Pale green	Very dim white	White	None	None

TABLE VI. *Potash feldspars.*

SPEC. NO.	LOCALITY	APPEARANCE	THERMOLUMINESCENCE	TRIBOLUM.	FLUORES.	PHOSPHORES. (Fe SPARK)
1	Kinkel Quarry, Bedford, N. Y.	Reddish xl. in quartz	Faint bluish white	Very faint white	None	None
2	Kinkel Quarry, Bedford, N. Y.	Black granular	Good white slightly bluish	None	None	Orange
3	Kinkel Quarry, Bedford, N. Y.	Dark red granular	Slightly fainter than No. 2	None	None	Strong orange
4	Kinkel Quarry, Bedford, N. Y.	Pale red granular	Like No. 3	None	None	Orange
5	Baylis Quarry, Bedford, N. Y.	Pale red massive	Like No. 3	None	None	None
6	Mt. Adam, N. Y.	Reddish massive	Same as No. 2	None	None	None
7	Riverside Drive, New York City	White	Faint bluish white	Very faint white	None	None
8	Golden Lake, Ontario	Dark red massive	Very faint bluish white	Orange	None	None

TABLE VII. *Other thermoluminescent minerals.*

SPEC. NO.	MINERAL	LOCALITY	APPEARANCE AND COLOR	THERMOLUM.	TRIBOLUM.	FLUOR. (ARGON BULB)	PHOS. (ARGON BULB)
1	Pectolite	Woodcliff, N. J.	Very coarse xls. white	Faint yellow white	Very strong yellow white	Buff (Fe spark)	Strong yellowish white (Fe spark)
2	Pectolite	Paterson, N. J.	Coarse xls. white	Faint yellow white	Strong yellowish white	—	—
3	Pectolite	Paterson, N. J.	White fibers	Faint white	White	—	—
4	Manganpectolite	Paterson, N. J.	Massive; reddish brown	Very faint white	—	Pale yellowish white (R. & M.+filter)	—
5	Amblygonite	San Diego, Co., Calif.	White	Faint bluish white	Faint bluish white	—	—
6	Amblygonite	Newry, Maine	Transp. white	Pale blue	White	—	—
7	Amblygonite	Black Hills, South Dakota	White	Strong light blue	—	—	—
8	Topaz	Utah	Sherry xl.	Faint orange	—	—	—
9	Tremolite	Easton, Pa.	White fibers	Faint bluish white	—	—	—
10	Hackmanite	Bancroft, Ontario	Purple or white	Strong yellowish green	—	Salmon	Bluish white
11	Willemite	Franklin, N. J.	Greyish white	Very faint greenish white	Faint greenish white	Strong green (Fe spark)	Strong green (Fe spark)
12	Calciumlarsenite	Franklin, N. J.	Greyish, massive	Faint greenish white (low temp.)	—	Bright lemon yellow (Fe spark)	Faint yellow (Fe spark)
13	Barylite	Franklin, N. J.	Grey massive	Bluish white	—	Blue (Fe spark)	—
14	Corundum, var. Ruby	North Carolina	Red xl.	Very faint low temp.	—	Red (R. & M. +filter)	—
15	Thulite	Fanny Gouge Mine, North Carolina	Pink xln.	White	—	—	—
16	Wollastonite	Essex County, N. Y.	White xln.	Faint bluish white	White	—	Red flash, then light yellow (Fe spark)
17	Sphalerite	Horn Silver Mine, Tintic Dist., Utah	Dark brown granular	Deep azure blue	Strong orange	Orange (green in spots)	Orange and green spots in greenish white ground
18	Moss agate	Fremont County, Wyoming	Faintly grey	Faint white	Strong white	Deep green	—
19	Chalcedony (pseudo after coral)	Ballast Pt., Tampa Bay, Florida	Black	Faint white	—	(R. & M. +filter): bright greenish yellow	—

cence in a microcline-perthite from Japan in which allanite and gadolinite were imbedded.

The orange phosphorescence of granular microcline from the Kinkel quarry is stronger in the partly altered dark red specimen 3 than in either the unaltered material of specimen 4, or the completely blackened specimen 2. It would seem that a certain exposure to the radioactivity of

included uranium minerals (or local concentration of rare earths) intensifies the phosphorescence, while stronger action accompanied by blackening tends to obscure it.

A pale bluish thermoluminescence of potash feldspar from France has been recently reported by Déribéré.[40] Two specimens furnished by him, one from Mt. Montchabert, Argentine (Savoie) and the other from St. Chely d'Apcher (Lozére) were tested and found to be essentially similar to those described above, the second being comparable to No. 2 in brilliance.

Other Thermoluminescent Minerals

A number of other thermoluminescent minerals are listed in Table VII. In the case of pectolite, both the depth of color and the intensity of triboluminescence increase with crystal size. The brilliance of triboluminescence in the very coarsely crystalline material from Woodcliff, New Jersey (No. 1) surpasses that of all other minerals tested with the exception of sphalerite from Tsumeb (Table VIII, specimen 3). The similarity in color of phosphorescence, triboluminescence and thermoluminescence in specimen 1 and of the last two in specimen 2 is worthy of note. Apparently, the partial alteration of pectolite with uptake of Mn (specimen 4) decreases the intensity of both thermo- and triboluminescence while increasing the fluorescence with respect to that of other specimens from the same locality.

The blue thermoluminescence of amblygonite which contains considerable fluorine does not in the least resemble the deeper blue commonly shown by fluorite. Although the similarity of tribo- and thermoluminescence in 5 is probably coincidence, there is also the possibility, as with other examples of color similarity, that this represents the normal condition, and that different colors in the various forms of luminescence of a specimen are due to modifying or inhibiting impurities. Judging from the frequency with which fluorine containing minerals such as fluorite, amblygonite, lepidolite, apatite, and topaz exhibit thermoluminescence, it appears that the presence of this element definitely favors the latter. This is probably also true of the tremolite and phlogopite here described, both of which commonly contain fluorine in small amounts.

A previously reported analysis[32] of hackmanite (specimen 10) showed the presence of small amounts of Li, Ga, Pb and Mn, together with traces of Be, Ba, Sr, Nd and P. The question of which ones are involved in the numerous luminescent properties* of this mineral and in its

[40] M. Déribéré, "Luminescent minerals; thermoluminescence of some feldspars and of a scapolite," Ann. Soc. Geol. Belg. **61**, Bull. No. 6 (1938).

* First described with special reference to thermoluminescence in a paper read by O. I. Lee before the Spring Meeting of the Optical Society of America, Corning, New York, March 4–6, 1937.

Table VIII. *Triboluminescent but not thermoluminescent.*

Spec. No.	Mineral	Locality	Natural Color	Triboluminescence	Fluor. (Argon Bulb)	Phos. (Argon Bulb)
1	Sphalerite	Jefferson City, Tenn.	Dark yellow	Fair white	—	—
2	Sphalerite	Lincoln Tunnel, Weehawken, N. J.	Translucent light yellow	Pale yellow white	—	—
3	Sphalerite	Tsumeb, South West Africa	Black metallic	Very strong yellow	Strong orange	Strong orange
4	Quartz xl.	Ellenville, N. Y.	Colorless	Strong yellow	—	—
5	Quartz (rose)	Bedford, N. Y.	Med. rose	Strong yellow	—	—
6	Willemite xl.	Franklin, N. J.	Opaque brown	Dim white	None—Argon bulb; strong green (Fe spark)	None
7	Willemite (grains)	Franklin, N. J.	Translucent green	Dim white	Strong green	None
8	Albite (Graphic Granite)	North Carolina	White	Fair white	—	—

TABLE IX. *Fluorescent or phosphorescent but neither thermoluminescent nor triboluminescent.*

SPEC. NO.	MINERAL	LOCALITY	NATURAL COLOR	FLUORESCENCE (ARGON BULB)	PHOSPHORESCENCE (ARGON BULB)
1	Hyalite	North Carolina	Transparent, colorless	Very strong green	None
2	Autunite	Yancey County, North Carolina	Pale greenish yellow	Very strong yellow green	None
3	Zircon	Statesville, N. Carolina	Dark brown	Strong yellow (R. & M. lamp and filter)	None
4	Selenite	Sicily	Transparent colorless	None	Medium white

reversible photosensitivity is open to conjecture.

The thermoluminescence of calcium larsenite (specimen 12) was the most unstable to heat of any of the minerals tested, extinction occurring before other minerals would have attained maximum brilliance. It contained only a trace of Mn. An analysis given by Palache, Bauer and Berman[41] showed 0.57 percent MnO, 0.48 percent FeO, and 0.23 percent MgO as minor impurities, Pb, Zn, Ca and Si being present in major amounts. After heating to faint redness, the fluorescence is somewhat fainter and definitely greener. This may indicate an admixture of willemite, the fluorescence of which, if present in small amount, would be masked by the brilliant yellow of the calcium larsenite until the latter had been partially destroyed by heat. However, since the greenish tinge after heating seemed to be uniformly distributed throughout the specimen, it may be that a portion of the calcium larsenite structure had been broken down with the formation of a willemite structure. The phosphorescence of calcium larsenite was almost completely destroyed by the above heat treatment.

It seems that only the phosphorescent variety of willemite (specimen 11) is thermoluminescent. The other specimens (numbers 6 and 7, Table VIII) lack thermoluminescence completely. The triboluminescence and thermoluminescence are somewhat similar in appearance, and, if stronger, would probably be more noticeably green, thus approaching the fluorescence and phosphorescence color. A spectrographic examination of barylite (specimen 13) showed the impurities to be several hundredths of a percent of Mg, and traces of Mn, Fe, Al, Pb, Zn and Sn in this order of decreasing concentration. They probably total only a tenth of a percent, which is in line with Palache's and Bauer's[42] original description of the mineral as being of unusually high purity. It would be difficult to say which of these impurities is responsible for the fluorescence and thermoluminescence of the mineral.

The sphalerite from Utah (specimen 17) is unique among specimens of that mineral in being strongly thermoluminescent.

SUPPLEMENTARY TYPES OF LUMINESCENCE

A number of the minerals tested were not thermoluminescent, but showed a noticeable triboluminescence, specimen 3 being by far the most brilliant in this respect. The prevalence of triboluminescence and the lack of thermoluminescence, even in strongly fluorescent and phosphorescent sphalerite, is somewhat surprising. Nelson[43] found by comparison of the triboluminescence spectrum of this mineral with that of the sparklets produced by grinding steel on an emery wheel, that the former consists of a selective emission of a thermal radiation band (Table VIII).

A few minerals were found which were strongly fluorescent or phosphorescent but neither thermoluminescent nor triboluminescent. These are given in Table IX. Haberlandt[1,25] has shown that the fluorescence in many hyalites is due to uranium, and, in zircons,[44] to rare earths.

[41] C. Palache, L. H. Bauer and H. Berman, "Larsenite and calcium larsenite, a new member of the chrysolite group from Franklin, N. J.," Am. Mineral. **13**, 142–44 (1928); "Larsenite, calcium larsenite, and the associated minerals at Franklin, N. J." *Ibid.* **13**, 334–40 (1928).

[42] C. Palache and L. H. Bauer, "On the occurrence of beryllium in the zinc deposits of Franklin, N. J." Am. Mineral. **15**, 30–33 (1930).

[43] D. M. Nelson, "Photographic spectra of triboluminescence," J. Opt. Soc. Am. **12**, 207–215 (1926).

[44] H. Haberlandt, "Fluorescence analysis of minerals," Mitt. d. Inst. f. Radiumforschung, No. 332, 11–13 (1934).

TABLE X. *Temperature of thermoluminescence.*

MINERAL	LOCALITY	TABLE NO. SPEC. NO.	FIRST VISIBLE WHITE	FIRST COLORED LUMINESCENCE	MAXIMUM BRILLIANCE	TOTAL EXTINCTION
Fluorite	Franklin, N. J.	Table I Spec. 1	44°C	77°C (greenish white)	213°C green	324°C
Calcite	Weehawken, N. J.	Table II Spec. 1	107°C	143°C yellowish white	246°C orange yellow	338°C
Fluorapatite	Hai Chow, China	Table III Spec. 2	70°C	102°C yellowish white	132°C greenish yellow	205°C just faintly visible; 288° extinction
Wernerite	Quebec	Table III Spec. 5	63°C	91°C yellowish white	213°C greenish yellow	413°C
Hackmanite	Bancroft, Ontario	Table VII Spec. 10	74°C	235°C greenish white	318°C yellowish green	402°C
Calcium Larsenite	Franklin, N. J.	Table VII Spec. 12	63°C	77°C pale greenish white	98°C pale greenish white	124°C

TABLE XA.

MINERAL	LOCALITY	TABLE NO. SPEC. NO.	FIRST VISIBLE WHITE	LOW TEMP. YELLOW	INTERMEDIATE BLUE	HIGH TEMP. YELLOW	HIGH TEMP. BLUE	TOTAL EXTINCTION
Fluorite	France	Table I Spec. 12	74°C	102°–232°C	232°–282°C	—	—	407°C
Fluorite	New Mexico	Table I Spec. 15	68°C	124°–180°C yellowish blue-white	180°–290°C	290°C–302°C yellow white	313°–463°C	474°C

The fluorescence of autunite is probably characteristic of the inherent structure, and closely resembles that of many uranium salts in general as pointed out by Iimori and Iwase.[45]

THE RELATION OF THERMOLUMINESCENCE TO TEMPERATURE

It was felt that the temperature at which various changes occur in the thermoluminescence of some of the foregoing minerals would be of interest, so a simple apparatus was assembled to make the necessary measurements. The most convenient source of heat found was a small electric hot plate designed for use with a Silex coffee maker. This heated up more quickly and uniformly than a larger laboratory plate. In order to cut out visible light from the resistor, it was covered with $\frac{1}{8}$-inch thick stainless steel plate, and a rheostat (rated at 17 ohms, 3.2 amp.) was placed in the circuit to control the rate of heating. An iron-constantan thermocouple with insulated leads and a welded hot junction about $\frac{1}{8}$-inch in diameter was used to measure the temperature. The lead wires were passed through a hole in the ornamental collar of the heater and bent down, so that the hot junction touched the plate. Several $\frac{1}{8}$-inch fragments of the mineral to be tested were then placed on the plate as close as possible to the hot junction, and a transparent silica crucible inverted over them. This last served to protect the specimen and hot junction from draughts, and to prevent loss of the mineral by decrepitation.

The thermocouple wires were connected to a Wilson-Maeulen microvoltmeter (range with iron-constantan couple: 0°–980°C), which was placed at a distance from the hot plate, behind a cardboard shield; so that the instrument could be illuminated for reading without lighting up the specimen. The readings in microvolts were converted to the corresponding temperatures by

[45] S. Iimori and E. Iwase, "The fluorescence spectrum of autunite," Sci. Papers Inst. Phys. Chem. Research (Tokyo) **34**, 372–6 (1938).

means of a calibration table furnished by the manufacturer.

All readings were made in a darkened room with the windows and doors closed to prevent draughts. One of the writers watched the specimen during heating and reported his observations to the other who read the microvoltmeter and recorded the corresponding condition of the thermoluminescence.

Temperatures in the neighborhood of 375°C (first faint redness of the steel plate) could be obtained with all the available resistance in the circuit. Above this, it was necessary to reduce the resistance by 4 to 8 ohms in order to reach the extinction point of the specimen within a reasonable time. Although no effort was made to control the duration of heating, its rate was kept as uniform as possible. The average time for a complete determination was about 5 minutes.

It must be admitted that the temperatures reported are somewhat low, due to cooling of the thermocouple hot junction by conduction of heat along the lead wires. However, unavoidable delays in observing and recording the state of luminescence at any given temperature, would tend to have the opposite effect. Since the color and relative intensity were only estimated visually, it is believed that the above-mentioned errors were not serious enough to detract much from the value of the results for comparative purposes (Table X). It will be noted that in general the first visible thermoluminescence, which appeared near or below boiling temperature, was white and that a definite interval (averaging about 30°C in the first 4 specimens) elapsed before the dominant color became visible. For most of the specimens, the temperature interval from first visibility to maximum brilliance was of about the same order of magnitude as that from maximum brilliance to extinction, the ratio varying between 1.5 to 1 and 1.5 to 2. For the hackmanite, however, this was 3 to 1.

Although its temperature of initial visibility was not unusually low, the thermoluminescence of calcium larsenite reached maximum brilliance, and then extinction far below that of any other mineral.

The similarity in color of thermoluminescence at various temperatures in these two fluorites (Table XA) would seem to indicate that similar activating impurities are present, (excepting the gap in the first above 290°C). The difference in intensity of the low temperature yellow is probably due to variation in the concentration of these impurities. The higher temperature yellow observed in the second specimen was much fainter than that given by other pieces from the same sample when heated more rapidly in a spoon. This may be due to non-uniformity of composition with respect to the activating impurities, or to the relative instability to heat of this particular component of the thermoluminescence. Four or five minutes were required to attain a temperature of 290°C on the hot plate as compared with one minute in the spoon.

Summary

(1). Visual observations were made on the thermoluminescence of 112 minerals comprising 32 species. The natural color, triboluminescence, fluorescence and phosphorescence, if any, of each are also described. Special attention was given the fluorites, calcites, apatites, lepidolites, and feldspars. A number of other halides, carbonates, phosphates, and micas related to the foregoing groups, but not thermoluminescent are mentioned. The triboluminescence of 8 nonthermoluminescent minerals (six species), and the fluorescence or phosphorescence of 4 others which were neither thermo- nor triboluminescent are described.

(2). Thermoluminescence appears to be most common in nonmetallic, anhydrous, crystalline minerals. The same characteristics plus a good cleavage are favorable for the production of triboluminescence. Seven species containing fluorine were found to be thermoluminescent, one specimen of fluorite out of 18 tested excepted.

(3). Most of the fluorites tested showed a strong thermoluminescence of several successive colors, which can probably be referred to the complex nature of the activating impurities. For convenience, the specimens have been divided into types based on the colors emitted. With one exception they were triboluminescent.

(4). The following general characteristics of thermoluminescence were observed in various fluorites:

(a) In common with all other minerals tested, thermoluminescence was completely destroyed by brief heating to redness.

(b) Following destruction by heat, attempts to regenerate the thermoluminescence of a fluorite by exposure to sun or ultraviolet light were unsuccessful.

(c) Tribothermoluminescence, or the regeneration of thermoluminescence by grinding, was observed in only one specimen. Apparently the phenomenon is not common in fluorites.

(d) A dim but relatively heat stable component, resembling triboluminescence, was observed in the strongly colored thermoluminescence of two specimens.

(e) A very strong green emission in one specimen was attributed to Mn which was not found in four specimens yielding a bluish or yellowish green followed by other colors.

(5). An intensified thermoluminescence was observed in a fluorite closely associated with the weakly radioactive mineral allanite, and similarly in the thermoluminescence and phosphorescence of a microcline containing radioactive gummite.

(6). The temperatures at which various changes occurred in thermoluminescence was measured in 8 specimens. Initial visibility was observed from 44° to 107°C and total extinction from 124° to 474°C.

(7). The triboluminescence of a number of minerals was unchanged after brief heating to redness. In several cases it was greatly diminished by grinding under water, confirming its partially electrical nature.

(8). Several points were noted which indicate that there is no simple relation between thermoluminescence and the other forms of luminescence studied:

(a) Any form of luminescence may exist alone in a mineral, or in conjunction with any of the others.

(b) A number of cases, particularly among calcites, were observed in which several factors appeared to act independently in each form of luminescence.

(c) In two fluorites, the relative heat stability of thermoluminescence, fluorescence, and triboluminescence, increased in that order. Several other instances of varying heat stability were noted.

(d) With the calcites no simple relationship was found between thermoluminescence intensity and reddish fluorescence, between an unusually brilliant initial thermoluminescence and triboluminescence, or between the Mn content and the intensity of yellow thermoluminescence.

(e) The few cases of similarity in the color of the various types of luminescence shown by a single specimen appeared to be exceptional. The opposite condition, probably arising from the presence of several activating or inhibiting impurities was more commonly observed. A plurality of impurities also accounts in part for (a), (b) and (d) above.

(9). Thermoluminescence of only one color, namely some shade of yellow, was observed in most calcites, with a less common bluish white constituting a second type. Only 15 percent of the specimens tested were triboluminescent. Four specimens, 3 of which were strongly thermoluminescent, contained traces of Pb and Sn.

(10). Although traces of Rb and Cs were found in three lepidolites, no connection with the characteristic orange red thermoluminescence was established.

Acknowledgments

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Optics in the Classroom

Experiment on Wien's Energy Distribution Law and Optical Pyrometry

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THE experiment described in this paper was developed to demonstrate, by simple and direct means, the fundamental dependence of optical pyrometry on Wien's energy distribution law.

The relative brightness of a self-luminous blackbody cavity is photometrically measured, in deep red-filtered light, in terms of that of a diffusely reflecting disk illuminated by a small movable incandescent lamp operated constantly at rated input. Cavity temperatures, which lie between 1000° and 1300°K, are measured with the aid of a calibrated thermocouple; brightness is expressed as the inverse square of the distance between the lamp filament and the disk. The optical system is similar to that of the conventional disappearing-filament pyrometer except that the filament is replaced by the illuminated disk and the filter transmits an unusually narrow visible spectral band. (See Fig. 1.)

The common logarithm of the brightness, as defined above, is plotted against the reciprocal of the absolute temperature, yielding a straight line from which the second radiation constant

$$c_2 = -2.30 \text{ (effective wave-length) (slope)}.$$

(See Fig. 2.)

Following this a complete calibration curve of the apparatus, as a pyrometer, is drawn by plotting theoretical brightness, const. $e^{-c_2/\lambda T}$ against temperature. A single careful measurement of the lamp-disk distance serves to determine the constant. This is illustrated by Fig. 3 in which experimental points obtained by several observers on different days have been placed on an adjusted theoretical curve.

The blackbody cavity is situated within a furnace constructed somewhat according to the design of Forsythe,[1,2] but with several simplifying modifications. A single Alundum tube, $1\frac{1}{2}$ inches inside diameter by 15 inches, is wound uniformly with Chromel A wire (cold resistance 58 ohms). After winding, the tube is coated with a layer of Alundum 1000° cement over which is wound a 30-ohm heater element having its turns concentrated near the ends. An annular disk winding of 9 ohms resistance is mounted concentrically around the window end to provide extra compensation for end loss. No. 20 wire is used throughout. The baffles and thermocouple junction are located approximately as shown in Forsythe's drawing. B and W type K30 firebrick insulation is employed.

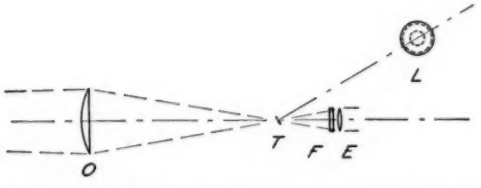

FIG. 1. Diagrammatic representation of the pyrometer optical system. *O*, objective; *T*, disk; *F*, filter; *E*, ocular; *L*, light source.

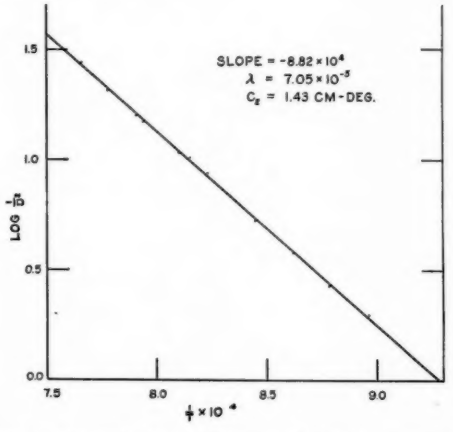

FIG. 2. Determination of the second radiation constant.

[1] E. P. Hyde and W. E. Forsythe, Astrophys. J. **51**, 244 (1920).
[2] W. E. Forsythe, *Measurement of Radiant Energy*, p. 27.

The telescope comprises an objective of 8 inches focal length and a $3\frac{1}{4}$-inch ocular, of spectacle quality, supported by Cenco optical bench clamps on a square steel bar.

The disk, made of thin sheet metal, is rendered diffusely reflecting by smoking over burning magnesium ribbon. Its diameter may be $\frac{1}{8}$ inch or less. It is mounted on the end of a stiff wire attached to a clamped rod.

The lamp is a 50-cp automobile headlight bulb contained in a metal housing having a circular window facing the disk. It is held by a Cenco clamp on a second optical bench which is placed at a convenient angle and directed so that the filament moves along a line passing through the disk when adjustments of brightness are made.

The unusually deep red filter, comprising dense blue cobalt glass and red celluloid (see Fig. 4) has the advantages that (a) the naturally very different color temperatures of the two fields are practically eliminated, (b) the effective wave-length of the light is nearly constant even at the lowest temperatures, and (c) the wave-length can be estimated closely by visual spectrometric examination of transmitted incandescent tungsten light without reference to any radiation law. The relatively low integral luminous intensities make it necessary to work in a dimly lighted room but otherwise present no difficulty.

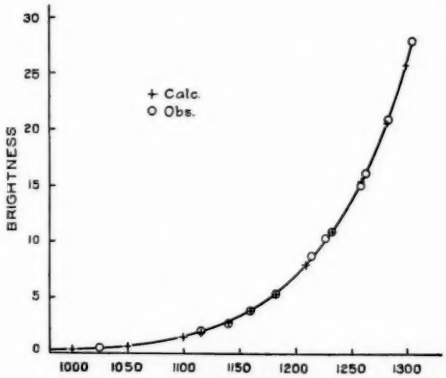

FIG. 3. Pyrometer calibration curve.

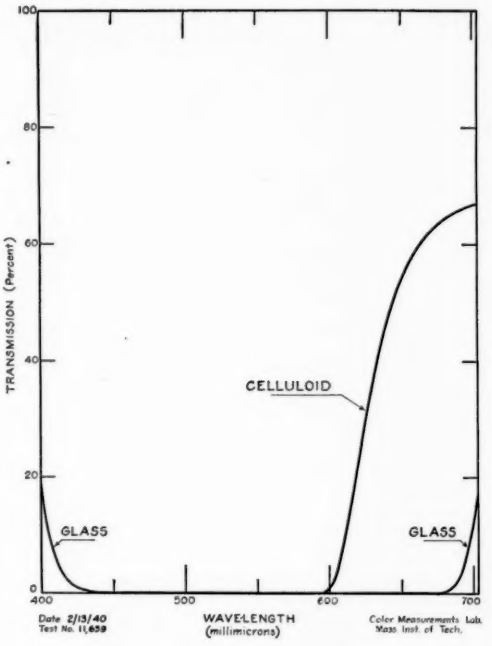

FIG. 4. Spectral transmission curves of the glass and celluloid components of the filter drawn by the Hardy color analyzer. (Courtesy of Dr. S. Q. Duntley.)

Because a relatively long time is required for re-establishment of thermal equilibrium after changing the furnace input, usually only three or four sets of measurements can be made during a three-hour laboratory period unless two or more furnaces are available. However, by selecting three temperatures so that low, intermediate, and high ranges are represented, each group of observers obtains data sufficient to determine the constant c_2 and to demonstrate the application of Wien's energy distribution law in the calibration of the optical apparatus when used as a pyrometer. The results of all groups, assigned different sets of temperatures, should be plotted on posted cumulative graphs, as in Figs. 2 and 3.

The Cathode-Ray Oscillograph as a Means of Demonstrating Elliptically Polarized Light

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THE cathode-ray oscillograph offers an interesting means of demonstrating the mathematical properties of polarized light. It will be shown in this paper that it is possible to produce on the screen of the oscillograph, patterns representing every possible state of polarization by merely manipulating three controls representing the three basic elements of a generalized ellipse, i.e., the magnitude of two perpendicular components and the phase difference between them. The underlying advantage in using electrical apparatus to produce these patterns lies in the fact that such equipment permits an easy adjustment of these basic elements in such a way that any one may be varied alone without disturbing the other two. Such control is very difficult to obtain with mechanical models or similar apparatus.

A beam of elliptically polarized light is usually most easily understood when it is considered as the resultant of the combination of two beams of plane polarized light, these components being oriented perpendicular to each other, and representing sinusoidal vibrations of the same frequency with a definite time lag or phase difference between corresponding points in the two waves. By purely electrical means it is possible to produce oscillograph patterns which possess geometrical properties identical to these same properties in a beam of polarized light, and, for the purposes of this demonstration, these patterns may be taken as true representations of certain states of polarization. For example, if a sinusoidally varying voltage, such as the 60-cycle lighting source, is impressed on the horizontal plates of the oscillograph, the "spot" will move rapidly back and forth to produce a horizontal line, the length of which is proportional to the magnitude of the impressed voltage. Likewise a vertical line of controlled length may be produced if the voltage is applied to the other set of deflecting plates. Considering the oscillograph screen as a cross section looking down into the beam of eliptically polarized light, then these two lines, when produced separately, may be taken as representations of the two basic perpendicularly plane polarized components of the ellipse. (Fig. 1, a and b.)

If the two components are now combined, and if they happen to be of equal magnitude and of the same phase (or "in step"), they will produce as a resultant pattern, a line at 45° to the two components, as shown in Fig. 1, c. If the magnitude of the two components remains the same, but their phase angle difference increases, the pattern on the screen successively changes to a left-handed ellipse with the major axis at 45° (1, d), then a left-handed circle when the phase difference becomes 90° (1, e). Further increasing of the phase difference results in a pattern consisting of a left-handed ellipse inclined at 135° (1, f) which degenerates into a straight line at 135° when this quantity approaches 180°. As the phase difference becomes greater than 180°,

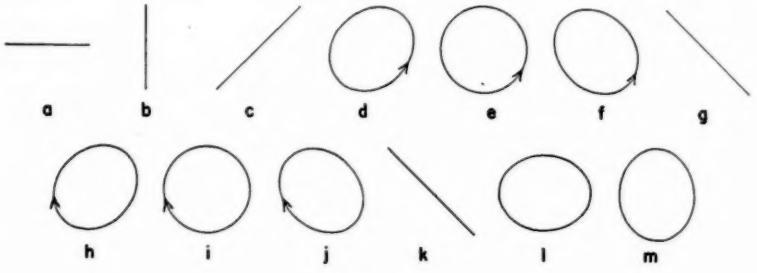

FIG. 1.

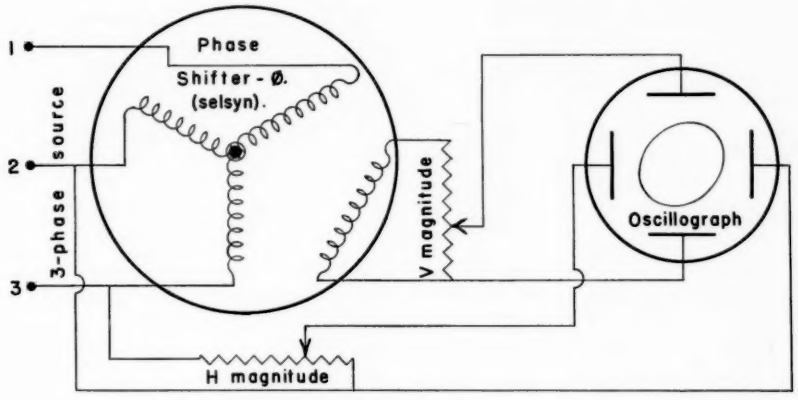

FIG. 2.

these patterns are then repeated in the reverse order exactly as before, except that the direction of rotation is now right-handed (Fig. 1, h to k).

If a beam of plane polarized light enters, for example, a Kerr cell or a photoelastic specimen under mechanical stress, and if the polarization azimuth is at 45° to the lines of electrical or mechanical stress, then this beam will undergo a continuous change as it penetrates deeper into the material. Neglecting absorption losses, it is apparent that these changes are caused entirely by a change in the relationship between the phases of the two components which are vibrating, respectively, parallel and perpendicular to the direction of stress, and the changes from patterns 1, c to 1, k will represent exactly what is happening to the beam as it penetrates the stressed material. This same series of patterns also represents the behavior of plane polarized light traversing any birefringent medium, whether it is naturally in this condition or only attains birefringency under the action of electrical, magnetic, or mechanical stress. Cases where the azimuth of incident polarization is at an angle of other than 45° to the lines of stress are similar to the above, except that the horizontal and vertical components would no longer be equal, and this inequality would distort the patterns so that they would still represent actual conditions in the polarized beam.

By altering the magnitude of one or the other of the two components with the phase difference set at 90° or 270°, it is possible to produce ellipses inclined at 0° or 90° (1, l and m), and, by varying all three elements, it is possible to represent any imaginable state of polarization, and to note directly the magnitude of each component and the phase difference.

The electrical equipment required to produce these patterns is quite simple and usually obtainable in most physics laboratories. The cathode-ray oscillograph is preferably of the electrostatic deflection type with a medium persistence screen. The most convenient way of producing two voltages whose phase difference may be varied continuously from 0° to 360° is to use a phase-changing transformer operated from a three-phase source. Self-synchronous motors, commonly used in pairs for data transmission or remote control of other apparatus, are readily adaptable to this purpose. Such motors are produced by leading American manufacturers under the tradenames "Selsyn," "Autosyn" or "Synchrosyn," and even the smallest of the commercial models is suitable for this experiment. The shaft should be provided with a 360° calibrated dial, a small knob or handle, and a light friction drag. The wiring diagram is shown in Fig. 2. A balanced three-phase source of the proper voltage is impressed on the Selsyn motor, which now acts as a transformer, and the output is connected through a potentiometer to the vertical-deflection plates of the oscillograph. One leg of the input is then connected, also through a potentiometer, to the horizontal-deflection plates. The latter voltage then acts as a reference or zero phase, while the phase of the vertical-deflection voltage is controlled by the rotation of the Selsyn motor shaft. It may be found impossi-

ble to obtain sufficiently large deflections on the oscillograph screen by this method without overvoltaging the Selsyn motor and, in this case, an oscillograph with built-in horizontal and vertical amplifiers is required. In any case, if the oscillograph is provided with amplifiers, the gain controls will serve the same purpose as the two potentiometers, and the latter may then be omitted.

It is a little more difficult to provide electrical means for showing the direction of rotation of the various patterns. One obvious method, if the three-phase supply is derived from a motor-generator set, is to slow down the frequency to about 15 cycles per second and the motion of the spot can then be seen directly. Another way is to use an oscillograph with a built-in sweep circuit which can be synchronized with the incoming voltage to produce some "hash" on top of the elliptical patterns, which will then appear to rotate very slowly. While this direction of rotation is not necessarily the true direction of rotation of the spot on the screen, nevertheless its direction does reverse as the phase angle passes through 0° and 180° and hence the transition from left-handed to right-handed states of polarization is made evident. No specific directions can be given for producing this "hash," since this depends too greatly upon the inherent characteristics of the oscillograph used.

With the equipment described it is apparent that the two gain controls, or potentiometers, determine the magnitude of the two perpendicular sinusoidal (or plane polarized) components, while the Selsyn motor alters the phase relationship of these two components without affecting the magnitude of either. In case Selsyn equipment is not available, a wound-rotor three-phase induction motor will serve the same purpose, although the frame size is usually inconveniently large. In localities in which only single-phase voltage is available, a phase-splitting network with Scott-connected transformers can be made to provide a reasonably balanced three-phase supply for the small power output demanded by this experiment, in case motor-generator equipment is not available.

Using an oscillograph having a three-inch diameter screen, a demonstration of the various generalized ellipse patterns was carried out simultaneously with the analogous polarized light experiments before a small laboratory section. With a five-inch oscillograph, but without any attempt to show left- and right-handed directions of rotation, this demonstration was also given in a large well-lighted lecture room seating over 200 students.

The author is indebted to Professor A. C. Hardy for his generous efforts in criticizing this manuscript.

Visual Teaching With Kodaslides

A NEW 28-page booklet, *Visual Teaching With Kodaslides*, of interest to educators who use visual methods in the classroom or lecture hall, has recently been published by the Eastman Kodak Company, Rochester.

Visual Teaching With Kodaslides discusses not only the manifold applications of the modern 2×2-inch slide, but also the technique of producing these slides, both in black-and-white and Kodachrome (full color). The text describes suitable cameras, enlarging, printing, and projection equipment; small scale and quantity production methods; and mounting, filing and storage procedure. A section is devoted to projection technique.

First printing of the new booklet has been mailed to a selected list of approximately 15,000 educators, museums, and other agencies concerned with visual teaching. Individuals with a legitimate interest in this field may request a personal copy from the Eastman Kodak Company, Rochester.